Nema opisa.

ZADATAK 8.1.1

Nacrtaj pravce čije su jednadžbe:

1) $x - 2y + 4 = 0$

2) $x + y - 1 = 0$

3) $2x + y - 3 = 0$

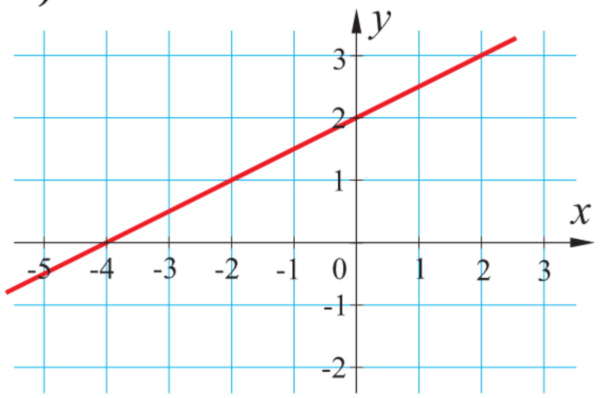
4) $2x - 3y + 6 = 0$

5) $3x + y = 0$

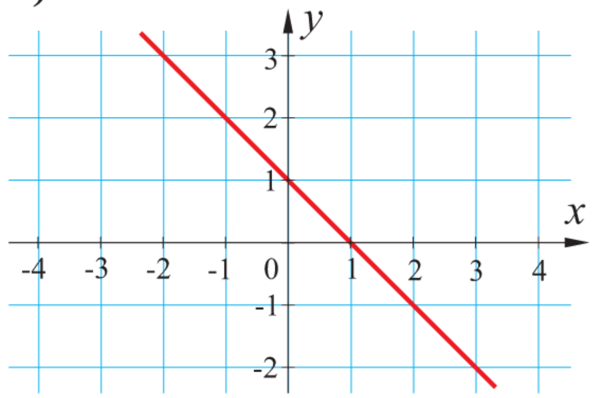
6) $3x - 2y - 4 = 0$.

RJEŠENJE

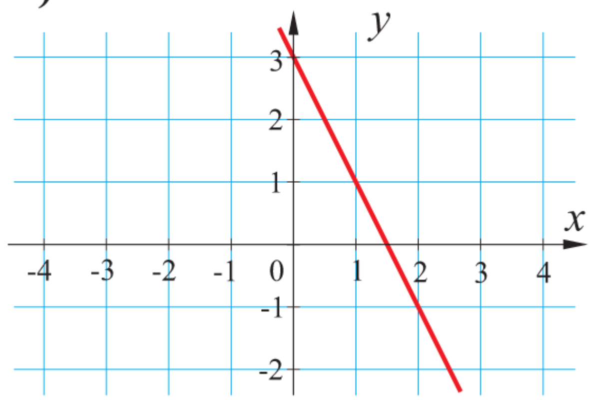
1)



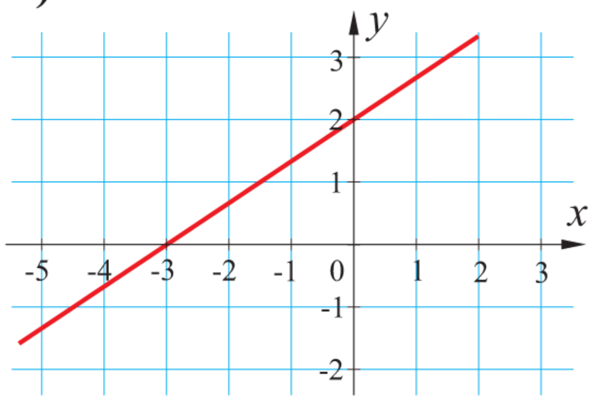
2)



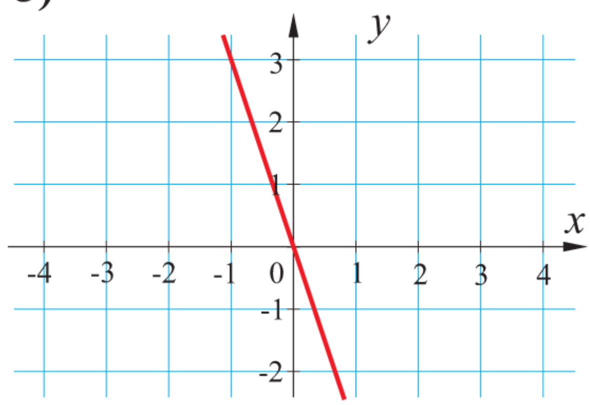
3)



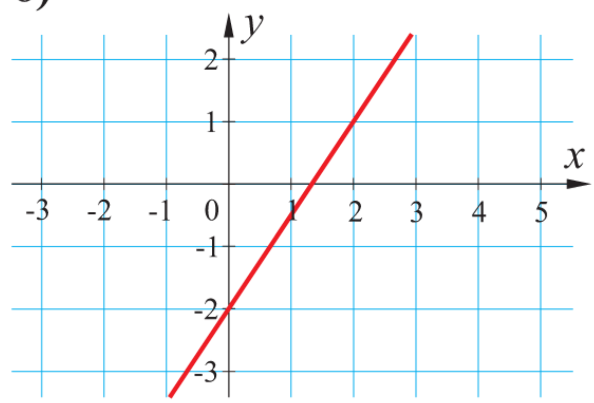
4)



5)



6)



ZADATAK 8.1.2

Nacrtaj pravce čije su jednadžbe:

1) $x - 1 = 0$

2) $y + 2 = 0$

3) $2x + 3 = 0$

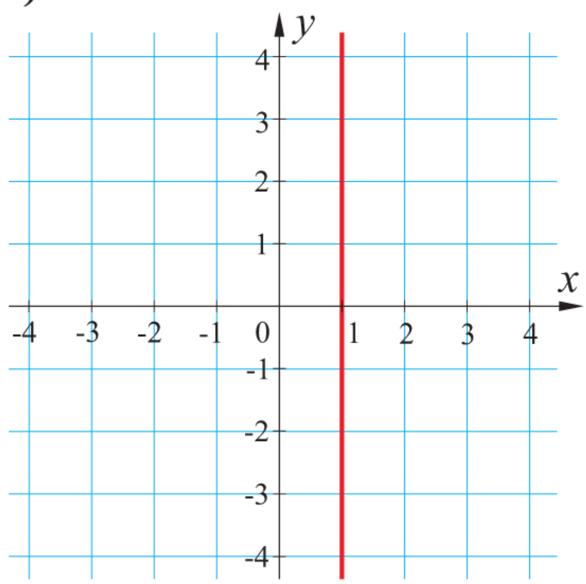
4) $3y + 6 = 0$

5) $2y - 11 = 0$

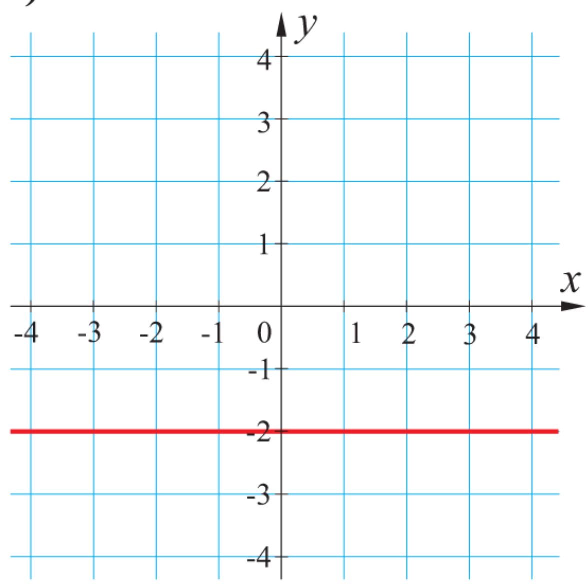
6) $4x + 12 = 0$.

RJEŠENJE

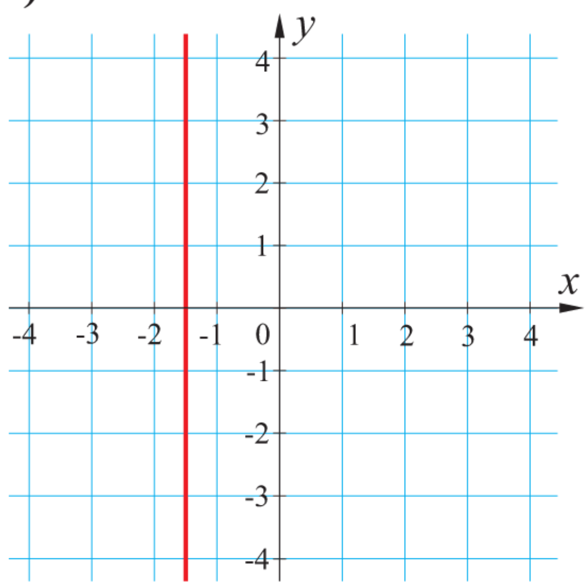
1)



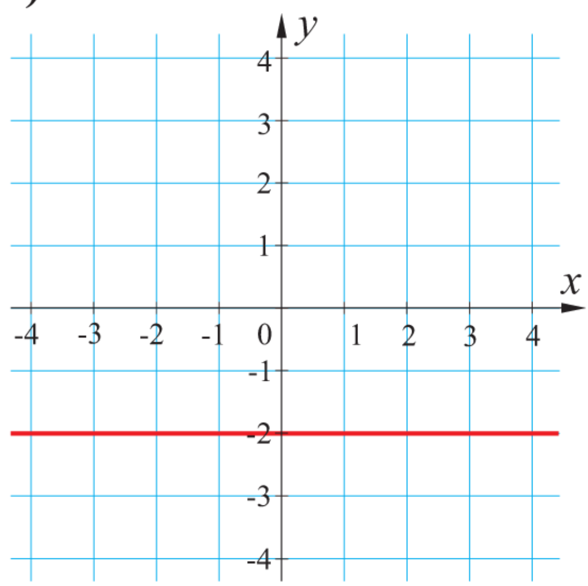
2)



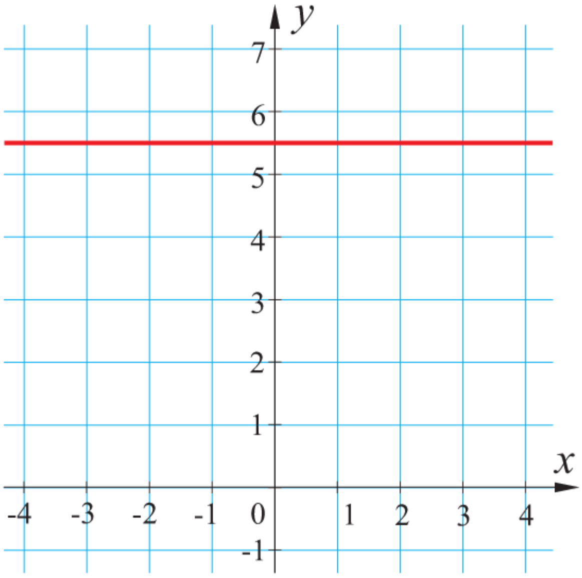
3)



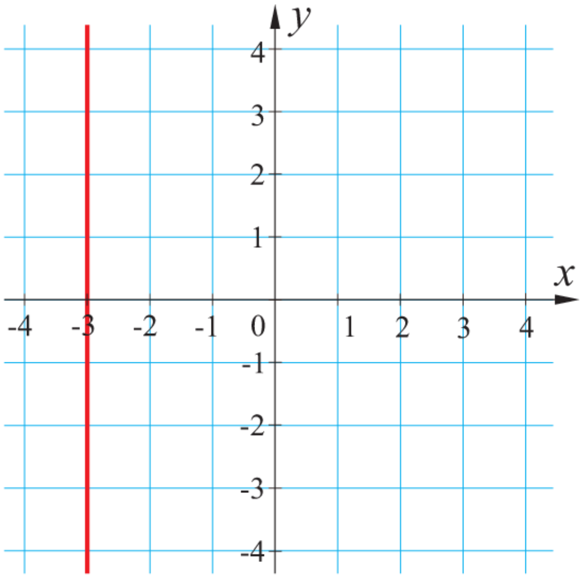
4)



5)



6)



ZADATAK 8.1.3

Nacrtaj pravce čije su jednadžbe:

1) $x - 3y + 6 = 0$

2) $2x - 3y = 0$

3) $x - 3 = 0$

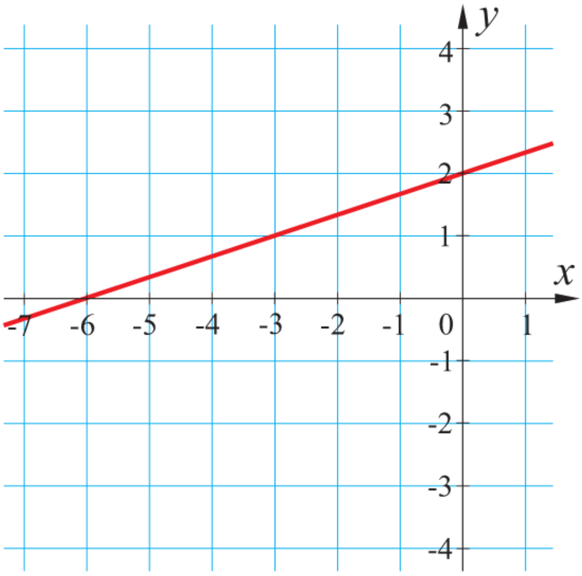
4) $x - 2y + 8 = 0$

5) $2y + 7 = 0$

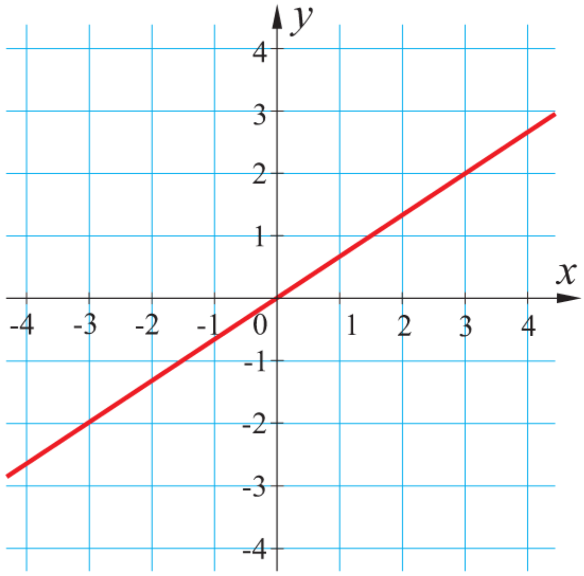
6) $x + 3y = 0.$

RJEŠENJE

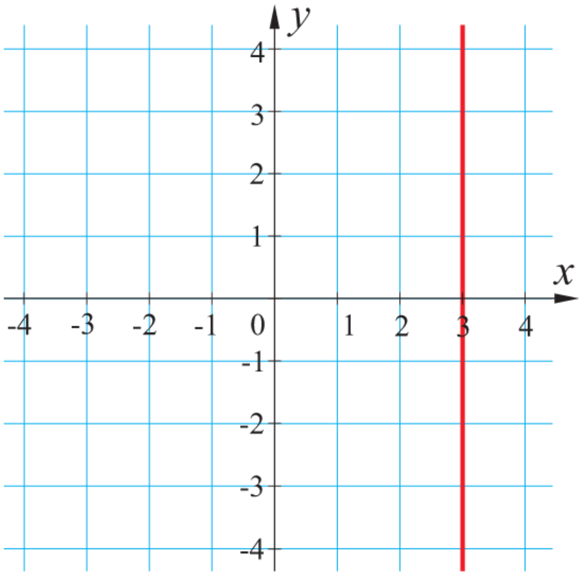
1)



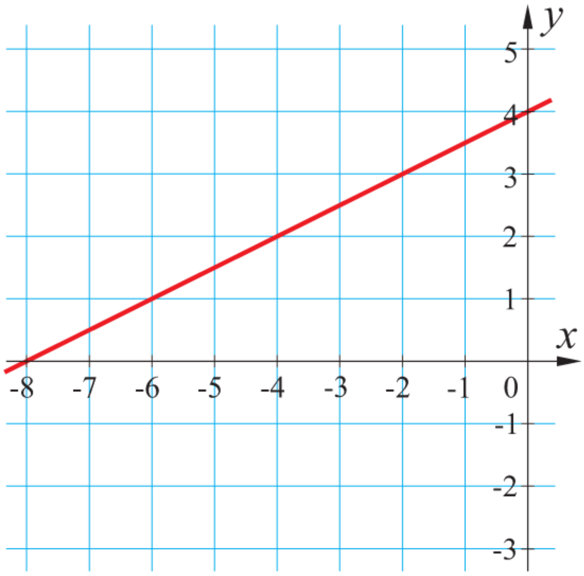
2)



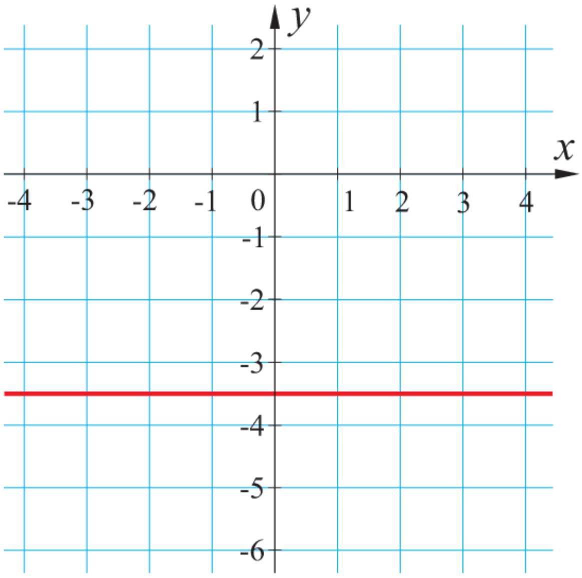
3)



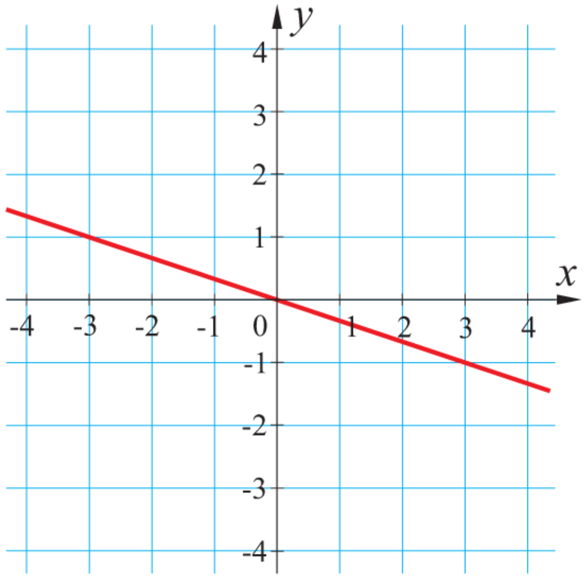
4)



5)



6)



ZADATAK 8.1.4

Jednadžbu pravca danu u implicitnom prevedi u eksplicitni oblik te odredi nagib pravca i odsječak na osi y:

1) $x + 2y - 6 = 0$

2) $4x + y = 0$

3) $x - 4y - 8 = 0$

4) $3x - y - 1 = 0$

5) $3x + 5y = 0$

6) $4x - 3y + 6 = 0$.

RJEŠENJE

1) $2y = -x + 6, \quad y = -\frac{1}{2}x + 3, \quad k = -\frac{1}{2}, \quad l = 3$

2) $y = -4x, \quad k = -4, \quad l = 0$

3) $4y = x - 8, \quad y = \frac{1}{4}x - 2, \quad k = \frac{1}{4}, \quad l = -2$

4) $y = 3x - 1, \quad k = 3, \quad l = -1$

5) $5y = -3x, \quad y = -\frac{3}{5}x, \quad k = -\frac{3}{5}, \quad l = 0$

6) $3y = 4x + 6, \quad y = \frac{4}{3}x + 2, \quad k = \frac{4}{3}, \quad l = 2.$

ZADATAK 8.1.5

Ucrtaj u koordinatnoj ravnini točke A i B. Odredi nagib pravca AB i kut što ga taj pravac zatvara s pozitivnim smjerom osi x ako je:

1) $A(-3, 3), \quad B(5, 7)$

2) $A(1, -2), \quad B(3, -3)$

3) $A(-2, 1), \quad B(4, 1)$

4) $A(2, 5), \quad B(2, -1)$

5) $A(-3, 2), \quad B(-1, -1)$

6) $A(1, -1), \quad B(4, 4)$.

RJEŠENJE

1) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{7-3}{5+3} = \frac{1}{2}$, $\operatorname{tg} \varphi = \frac{1}{2}$, $\varphi = 26^\circ 34'$

2) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{-3+2}{3-1} = -\frac{1}{2}$, $\operatorname{tg} \varphi = -\frac{1}{2}$, $\varphi = -26^\circ 34'$

3) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{1-1}{4+2} = 0$, $\operatorname{tg} \varphi = 0$, $\varphi = 0^\circ$

4) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{-1-5}{2-2} = \frac{-6}{0}$, nagib nije definiran, pravac je paralelan s y-osi, $\varphi = 90^\circ$

5) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{-1-2}{-1+3} = -\frac{3}{2}$, $\operatorname{tg} \varphi = -\frac{3}{2}$, $\varphi = -56^\circ 19'$

6) $k = \frac{y_B - y_A}{x_B - x_A} = \frac{4+1}{4-1} = \frac{5}{3}$, $\operatorname{tg} \varphi = \frac{5}{3}$, $\varphi = 59^\circ 2'$.

ZADATAK 8.1.6

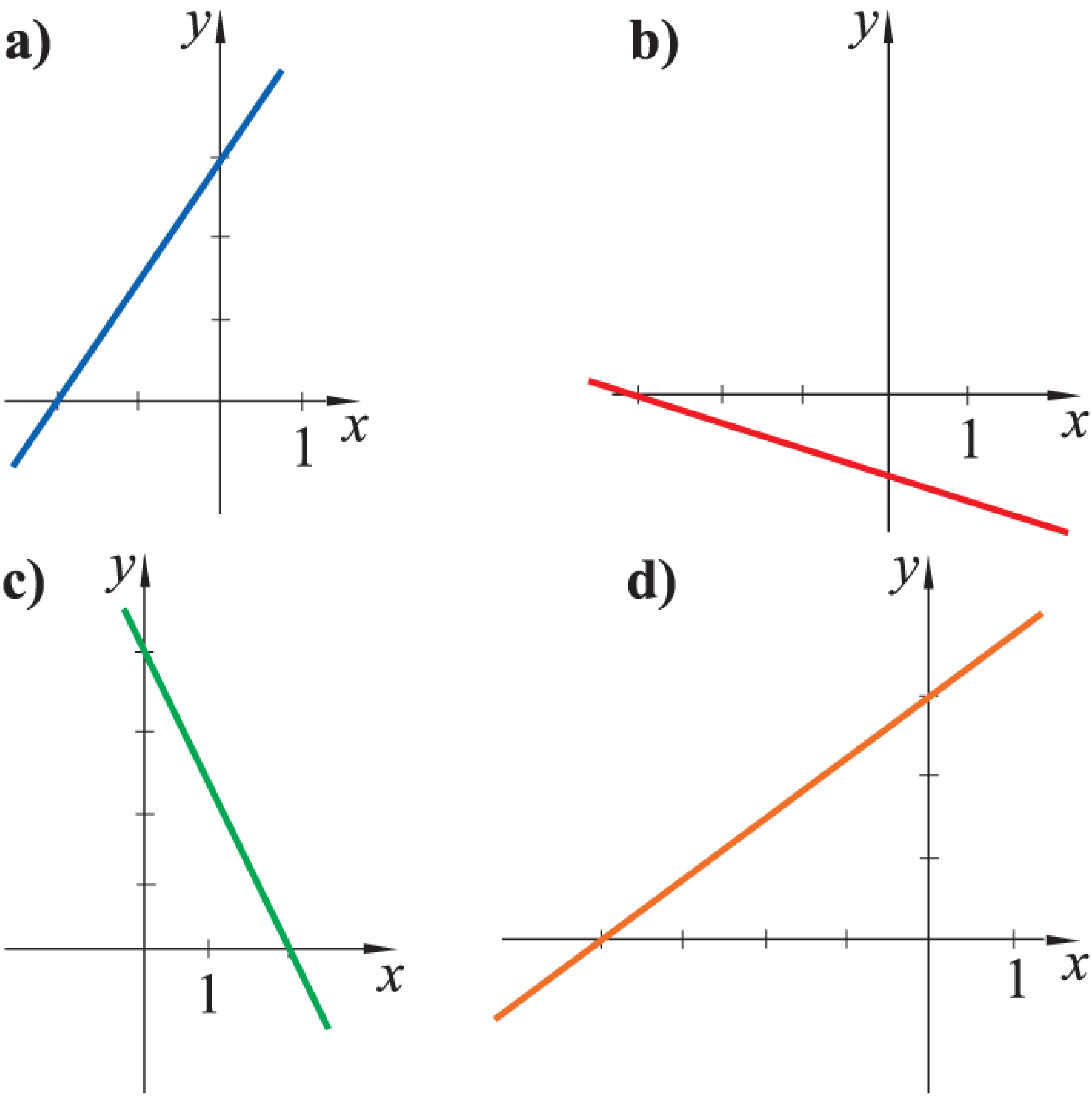
Koje jednadžbe pripadaju pojedinom pravcu:

1) $2x + y - 4 = 0$

2) $3x - 2y + 6 = 0$

3) $3x - 4y + 12 = 0$

4) $x + 3y + 3 = 0$?



RJEŠENJE

1) $y = 0 \;\Rightarrow\; x = 2$ pod c)

2) $y = 0 \;\Rightarrow\; x = -2$ pod a)

3) $y = 0 \;\Rightarrow\; x = -4$ pod d)

4) $y = 0 \;\Rightarrow\; x = -3$ pod b).

ZADATAK 8.1.7

Napiši jednadžbu pravca koji prolazi točkama A i B ako je:

1) $A(0, 5)$, $B(1, 1)$

2) $A(-2, 2)$, $B(3, -3)$

3) $A(1, 4)$, $B(5, 4)$

4) $A(-3, -2)$, $B(1, 0)$

5) $A(1, -2)$, $B(4, -11)$

6) $A(-3, 20)$, $B(-3, -8)$.

RJEŠENJE

1) $A(0, 5)$, $B(1, 1)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 5 &= \frac{1 - 5}{1 - 0}(x - 0) \\
y - 5 &= \frac{-4}{1} \cdot x \\
y &= -4x + 5 \rightarrow \text{eksplicitni oblik} \\
4x + y - 5 &= 0 \rightarrow \text{implicitni oblik}
\end{aligned}
$$

2) $A(-2, 2)$, $B(3, -3)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 2 &= \frac{-3 - 2}{3 + 2}(x + 2) \\
y - 2 &= \frac{-5}{-5} \cdot (x + 2) \\
y - 2 &= -x - 2 \\
y &= -x \rightarrow \text{eksplicitni oblik} \\
x + y &= 0 \rightarrow \text{implicitni oblik}
\end{aligned}
$$

3) $A(1, 4)$, $B(5, 4)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 4 &= \frac{4 - 4}{5 - 1}(x - 1) \\
y - 4 &= 0 \rightarrow \text{implicitni oblik} \\
y &= 4 \rightarrow \text{eksplicitni oblik}
\end{aligned}
$$

4) $A(-3, -2)$, $B(1, 0)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 2 = \frac{0 + 2}{1 + 3}(x + 3)$$

$$y + 2 = \frac{2}{4} \cdot (x + 3)$$

$$y = \frac{1}{2}x + \frac{3}{2} - 2$$

$$y = \frac{1}{2}x - \frac{1}{2} \rightarrow \text{eksplicitni oblik}$$

$$2y = x - 1$$

$$x - 2y - 1 = 0 \rightarrow \text{implicitni oblik}$$

5) $A(1,\ -2)$, $B(4,\ -11)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 2 = \frac{-11 + 2}{4 - 1}(x - 1)$$

$$y + 2 = \frac{-9}{3} \cdot (x - 1)$$

$$y + 2 = -3x + 3$$

$$y = -3x + 1 \rightarrow \text{eksplicitni oblik}$$

$$2y = x - 1$$

$$3x + y - 1 = 0 \rightarrow \text{implicitni oblik}$$

6) $A(-3, 20)$, $B(-3,\ -8)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 20 = \frac{-8 - 20}{-3 + 3}(x + 3)$$

$$y - 20 = \frac{-28}{0} \cdot (x + 3) \implies \text{nije definiran nagib pravca}$$

$$\implies = x + 3 = 0$$

$$x = -3 \rightarrow \text{eksplicitni oblik}$$

$$2y = x - 1$$

$$x + 3 = 0 \rightarrow \text{implicitni oblik}$$

ZADATAK 8.1.8

Provjeri pripadaju li točke A, B i C jednom pravcu:

1) $A(-3, -4), B(1, -1), C(5, 2)$

2) $A(1, 3), B(-2, 5), C(7, -1)$

3) $A(-7, 14), B(3, -2), C(-2, 6)$.

RJEŠENJE

1) $A(\overset{x_1}{-3}, \overset{y_1}{-4}), B(\overset{x_2}{1}, \overset{y_2}{-1}), C(5, 2)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y + 4 &= \frac{-1 + 4}{1 + 3}(x + 3) \\
y + 4 &= \frac{3}{4}(x + 3) \\
y + 4 &= \frac{3}{4}x + \frac{9}{4} \quad / \cdot 4 \\
4y + 16 &= 3x + 9 \\
3x - 4y - 7 &= 0 \\
3 \cdot 5 - 4 \cdot 2 - 7 &= 15 - 8 - 7 = 0 \implies C \in AB
\end{aligned}
$$

2) $A(\overset{x_1}{1}, \overset{y_1}{3}), B(\overset{x_2}{-2}, \overset{y_2}{5}), C(7, -1)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 3 &= \frac{5 - 3}{-2 - 1}(x - 1) \\
y - 3 &= \frac{2}{-3}(x - 1) \\
y - 3 &= -\frac{2}{3}x + \frac{2}{3} \quad / \cdot 3 \\
3y - 9 &= -2x + 2 \\
2x + 3y - 11 &= 0 \\
2 \cdot 7 + 3 \cdot (-1) - 11 &= 14 - 3 - 11 = 0 \implies C \in AB
\end{aligned}
$$

3) $A(\overset{x_1}{-7}, \overset{y_1}{14}), B(\overset{x_2}{3}, \overset{y_2}{-2}), C(-2, 6)$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 14 &= \frac{-2 - 14}{3 + 7}(x + 7) \\
y - 14 &= \frac{-16}{10}(x + 7) \\
y - 14 &= -\frac{8}{5}x - \frac{56}{5} \quad / \cdot 5 \\
5y - 70 &= -8x - 56 \\
8x + 5y - 14 &= 0 \\
8 \cdot (-2) + 5 \cdot 6 - 14 &= -16 + 30 - 14 = 0 \implies C \in AB
\end{aligned}
$$

ZADATAK 8.1.9

Točke $A(-1, y)$, $B(-3, 3)$, $C(5, -1)$ pripadaju jednom pravcu. Odredi ordinatu točke A.

RJEŠENJE

$A(-1, y),\ B(\overset{x_1}{-3},\ \overset{y_1}{3}),\ C(\overset{x_2}{5},\ \overset{y_2}{-1})$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 3 &= \frac{-1-3}{5+3}(x+3) \\
y - 3 &= \frac{-4}{8}(x+3) \\
y - 3 &= -\frac{1}{2}x - \frac{3}{2} \quad / \cdot 2 \\
2y - 6 &= -x - 3 \\
x + 2y - 3 &= 0 \\
-1 + 2 \cdot y - 3 &= 0 \\
2 \cdot y &= 4 \\
y &= 2
\end{aligned}
$$

ZADATAK 8.1.10

Točke $A(-1, 2)$, $B(x, 4)$ i $C(5, 6)$ pripadaju jednom pravcu. Iz tog uvjeta odredi apscisu točke B.

RJEŠENJE

$A(\overset{x_1}{-1},\ \overset{y_1}{2}),\ B(x, 4)$ i $C(\overset{x_2}{5},\ \overset{y_2}{6})$

$$
\begin{aligned}
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
y - 2 &= \frac{6-2}{5+1}(x+1) \\
y - 2 &= \frac{4}{6}(x+1) \\
y - 2 &= \frac{2}{3}x + \frac{2}{3} \quad / \cdot 3 \\
3y - 6 &= 2x + 2 \\
2x - 3y + 8 &= 0 \\
2 \cdot x - 3 \cdot 4 + 8 &= 0 \\
2 \cdot x - 12 + 8 &= 0 \\
2 \cdot x &= 4 \\
x &= 2
\end{aligned}
$$

ZADATAK 8.1.11

Točkom $A(-1, 2)$ položi pravac s koeficijentom smjera $-\frac{2}{3}$.

RJEŠENJE

$$A(-1, 2)$$
$$k = -\frac{2}{3}$$
$$y - y_1 = k(x - x_1)$$
$$y - 2 = -\frac{2}{3}(x + 1)$$
$$y - 2 = -\frac{2}{3}x - \frac{2}{3} \quad / \cdot 3$$
$$3y - 6 = -2x - 2$$
$$2x + 3y - 4 = 0$$

ZADATAK 8.1.12

Točkom $A(3, -2)$ položi pravac koji os ordinata siječe u točki $B(0, -3)$.

RJEŠENJE

$$A(3, -2),\ B(0, -3) \Rightarrow \quad l = -3$$
$$y = kx + l \Rightarrow \quad -2 = k \cdot 3 - 3$$
$$1 = 3k$$
$$k = \frac{1}{3}$$

$$y = \frac{1}{3}x - 3$$
$$3y = x - 9$$
$$x - 3y - 9 = 0$$

ZADATAK 8.1.13
Kako glasi jednadžba pravca simetričnog pravcu $2x + 3y - 7 = 0$ s obzirom na

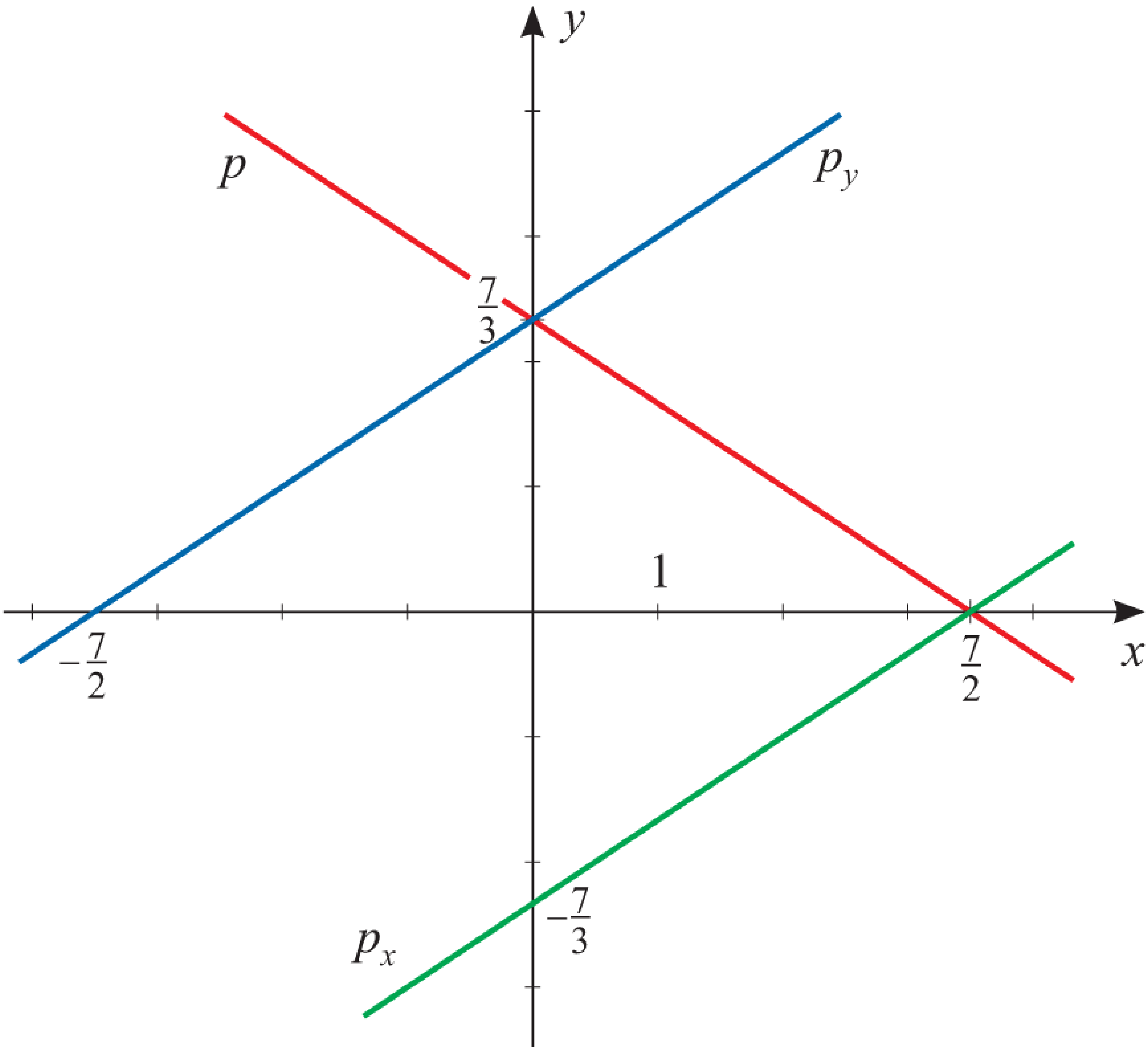
1) os apscisa

2) os ordinata?

RJEŠENJE

$$2x + 3y - 7 = 0$$
$$3y = -2x + 7 \quad / : 3$$
$$y = -\frac{2}{3}x + \frac{7}{3}$$

x	y
0	$\frac{7}{3}$
$\frac{7}{2}$	0



1) p_x prolazi kroz točke $\left(\frac{7}{2}, 0\right)$, $\left(0, -\frac{7}{3}\right)$

$$\Rightarrow \quad \frac{x}{m} + \frac{y}{n} = 1$$

$$\frac{x}{\frac{7}{2}} + \frac{y}{-\frac{7}{3}} = 1$$

$$\frac{2x}{7} - \frac{3y}{7} = 1 \quad / \cdot 7$$

$$2x - 3y = 7$$

$$2x - 3y - 7 = 0$$

2) p_y prolazi kroz točke $\left(-\frac{7}{2}, 0\right)$, $\left(0, \frac{7}{3}\right)$

$$\Rightarrow \quad \frac{x}{m} + \frac{y}{n} = 1$$

$$\frac{x}{-\frac{7}{2}} + \frac{y}{\frac{7}{3}} = 1$$

$$-\frac{2x}{7} + \frac{3y}{7} = 1 \quad / \cdot 7$$

$$-2x + 3y = 7$$

$$-2x + 3y - 7 = 0 \quad / \cdot (- 1)$$

$$2x - 3y + 7 = 0$$

ZADATAK 8.1.14

Odredi jednadžbu pravca koji je simetričan pravcu $3x - 4y + 8 = 0$ s obzirom na

1) os apscisa

2) os ordinata.

Nacrtaj sliku.

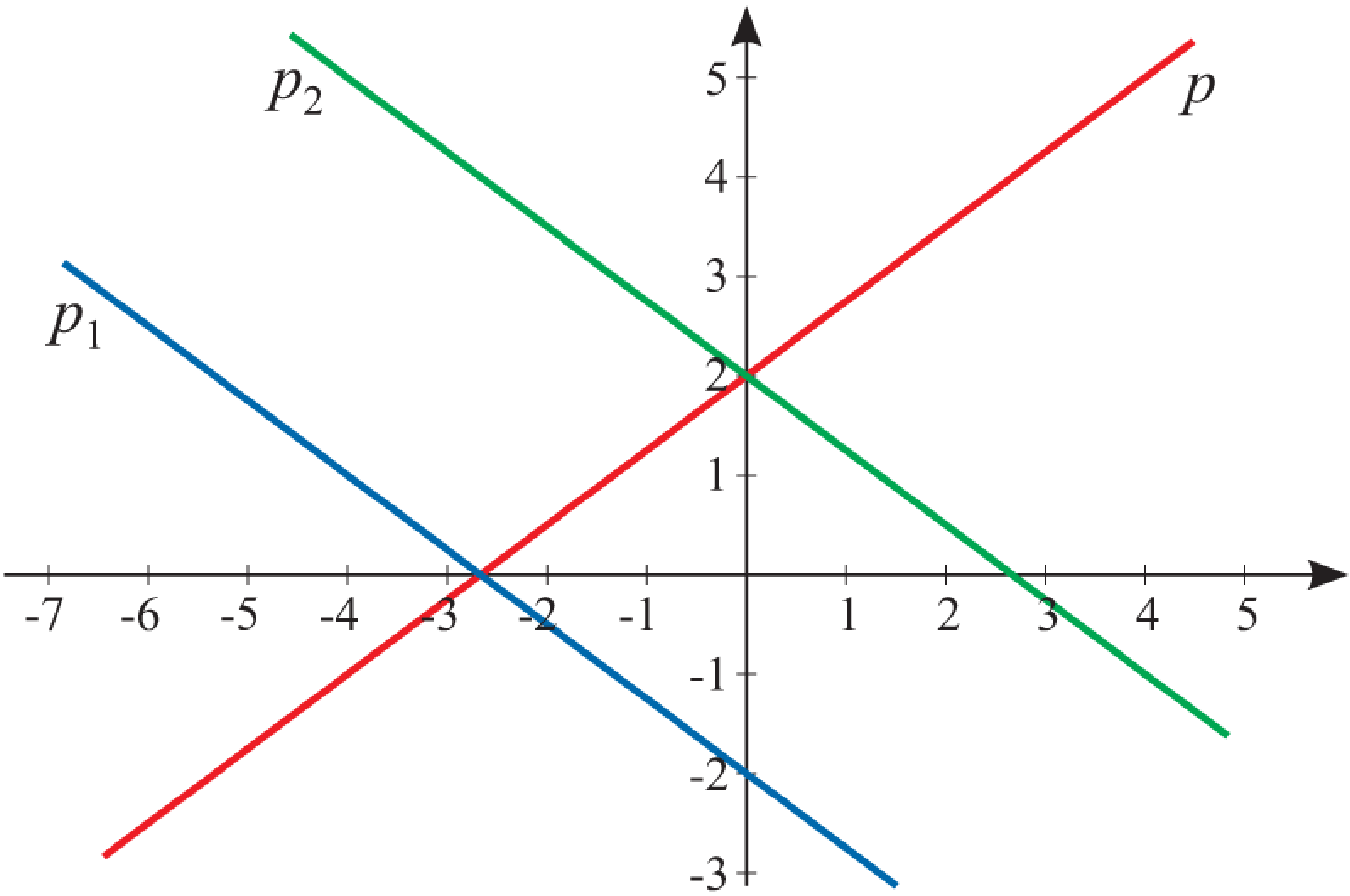
RJEŠENJE

1) $A(x, y) \in p \implies A_1(x, -y) \in p_1$

$$\implies p_1 \ldots \quad 3x - 4 \cdot (-y) + 8 = 0$$
$$3x + 4y + 8 = 0$$

2) $A(x, y) \in p \implies A_2(-x, y) \in p_2$

$$\implies p_2 \ldots \quad 3 \cdot (-x) - 4y + 8 = 0$$
$$-3x - 4y + 8 = 0$$
$$3x + 4y - 8 = 0$$



ZADATAK 8.1.15
Kako glasi jednadžba pravca koji je simetričan pravcu $x-3y=0$ s obzirom na pravac

1) $x-y=0$

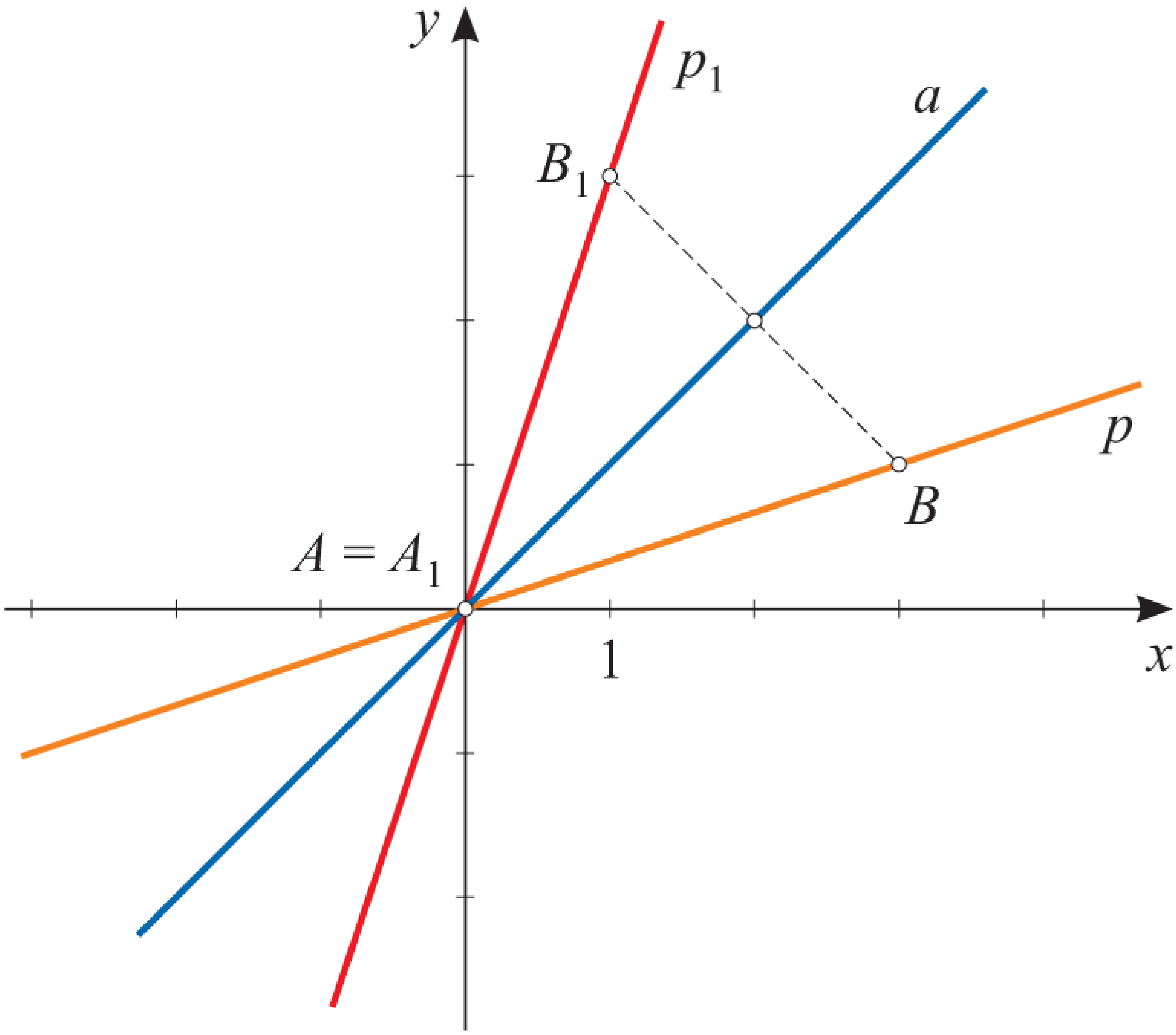
2) $x+y=0$?

RJEŠENJE

$$x-3y=0$$
$$3y=x \quad /:3$$
$$p \quad \ldots \quad y=\frac{1}{3}x$$

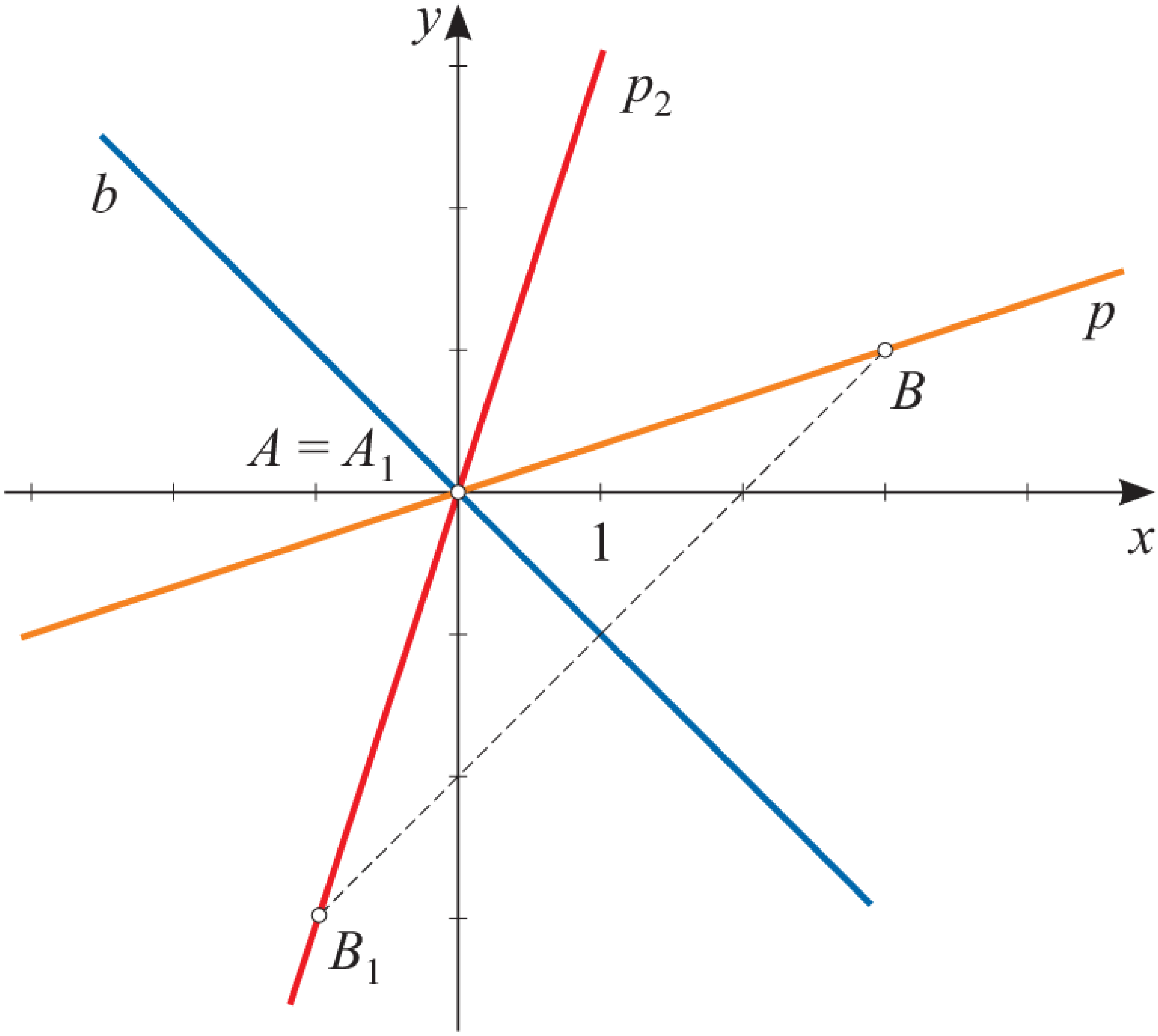
x	y
0	0
3	1

1) $a \ \ldots \ x-y=0,\ x=y$



$$\left.\begin{array}{l} A(0,0)\in p \implies A_1(0,0)\in p_1 \\ B(3,1)\in p \implies B_1(1,3)\in p_1 \end{array}\right\} \implies \begin{array}{c} p_1 \ldots y-0=\frac{3-0}{1-0}(x-0) \\ y=3x \\ 3x-y=0 \end{array}$$

2) $b \ \ldots \ x+y=0,\ y=-x$



$$\left.\begin{array}{r} A(0,0)\in p \;\Rightarrow\; A_2(0,0)\in p_2 \\ B(3,1)\in p \;\Rightarrow\; B_2(-1,-3)\in p_2 \end{array}\right\} \Rightarrow \begin{array}{l} p_2 \ldots y-0=\dfrac{-3-0}{-1-0}(x-0) \\ \quad y=-3x \\ \quad 3x-y=0 \end{array}$$

ZADATAK 8.1.17

Napiši jednadžbu pravca koji prolazi točkom $T(2, 2)$, a s pozitivnim dijelom osi apscisa zatvara dvostruko veći kut od pravca $y = 3x + 4$.

RJEŠENJE

$$a \ldots \; T(\overset{x_1}{2}, \overset{x_2}{2}), \quad 2\alpha, \quad k_a$$

$$b \ldots \; y = 3x + 4, \quad \alpha \Longrightarrow \operatorname{tg} \alpha = 3 = k_b$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} = \frac{2 \cdot 3}{1 - 9} = \frac{6}{-8} = -\frac{3}{4} \Longrightarrow k_a = -\frac{3}{4}$$

$$b \ldots \; y - y_1 = k_a(x - x_1)$$

$$y - 2 = -\frac{3}{4}(x - 2)$$

$$y - 2 = -\frac{3}{4}x + \frac{3}{2} \quad / \cdot 4$$

$$4y - 8 = -3x + 6$$

$$b \ldots \; 3x + 4y - 14 = 0$$

ZADATAK 8.1.18

Dva paralelna pravca, $2x - 5y + 6 = 0$ i $2x - 5y - 7 = 0$ dijele ravninu na tri područja: prugu između njih te na dvije poluravnine. Kojem od tih triju područja pripadaju točke: $A(2, 1)$, $B(3, 0)$, $C(-2, 2)$, $D(10, 3)$, $E(-5, -3)$?

RJEŠENJE

$p \; \ldots \; 2x - 5y + 6 = 0$

$q \; \ldots \; 2x - 5y - 7 = 0$

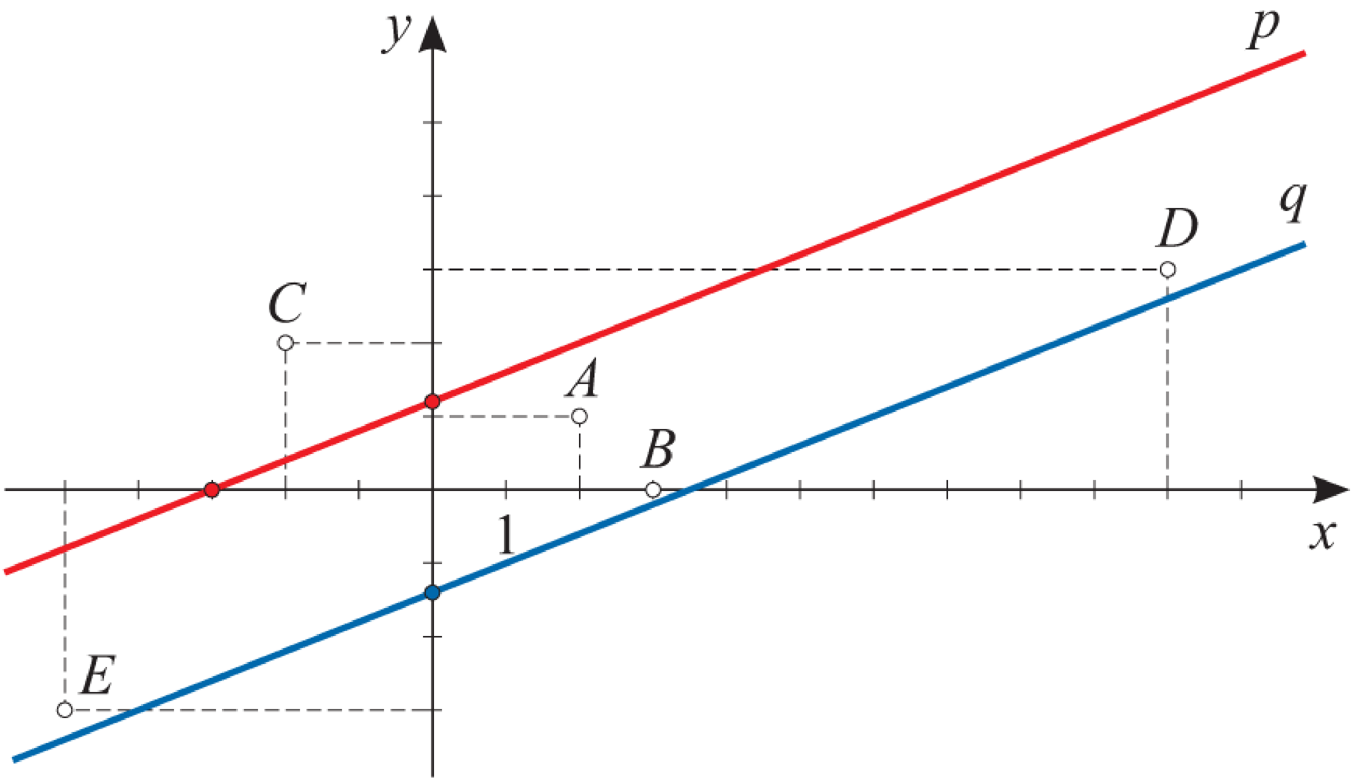
–

$p \; \ldots \; y = \frac{2}{5}x + \frac{6}{5}$

x	y
0	$\frac{6}{5}$
-3	0

$q \; \ldots \; y = \frac{2}{5}x - \frac{7}{5}$

x	y
0	$-\frac{7}{5}$
$\frac{7}{2}$	0



Za ordinatu točaka koje pripadaju donjoj poluravnini vrijedi $y < \frac{2}{5}x - \frac{7}{5}$. Za ordinatu točaka koje pripadaju pruzi vrijedi $\frac{2}{5}x - \frac{7}{5} < y < \frac{2}{5}x + \frac{6}{5}$. Za ordinatu točaka koje pripadaju gornjoj poluravnini vrijedi $y > \frac{2}{5}x - \frac{7}{5}$.

$$A(2, 1) \; \ldots \; 1 = \frac{2}{5} \cdot 2 + \frac{1}{4}, \quad -\frac{7}{5} < \frac{1}{4} < \frac{6}{5} \;\Rightarrow\; A \text{ pruga}$$

$$B(3, 0) \; \ldots \; 0 = \frac{2}{5} \cdot 3 - \frac{6}{5}, \quad -\frac{7}{5} < -\frac{6}{5} < \frac{6}{5} \;\Rightarrow\; B \text{ pruga}$$

$$C(-2, 2) \; \ldots \; 2 = \frac{2}{5} \cdot (-2) + \frac{14}{5}, \quad \frac{14}{5} > \frac{6}{5} \;\Rightarrow\; C \text{ gornja poluravnina}$$

$$D(10, 3) \; \ldots \; 3 = \frac{2}{5} \cdot 10 - \frac{5}{5}, \quad -\frac{7}{5} < -\frac{5}{5} < \frac{6}{5} \;\Rightarrow\; D \text{ pruga}$$

$$E(-5, -3) \; \ldots \; -3 = \frac{2}{5} \cdot (-5) - \frac{5}{5}, \quad -\frac{7}{5} < -\frac{5}{5} < \frac{6}{5} \;\Rightarrow\; E \text{ pruga}$$

ZADATAK 8.1.19

Stranice trokuta ABC leže na pravcima $2x-9y-30=0$, $2x-y+2=0$ i $6x+5y-26=0$. Koje od navedenih točaka leže unutar trokuta ABC: $D(-1,-2)$, $E(2,3)$, $F(5,0)$, $G(4,1)$, $H(5,-1)$?

RJEŠENJE

$$a \ \dots \ 2x-9y-30=0$$
$$b \ \dots \ 2x-y+2=0$$
$$c \ \dots \ 6x+5y-26=0$$

$$a \cap b \qquad \left.\begin{array}{r} 2x-9y-30=0 \\ 2x-y+2=0 \end{array}\right\} -$$

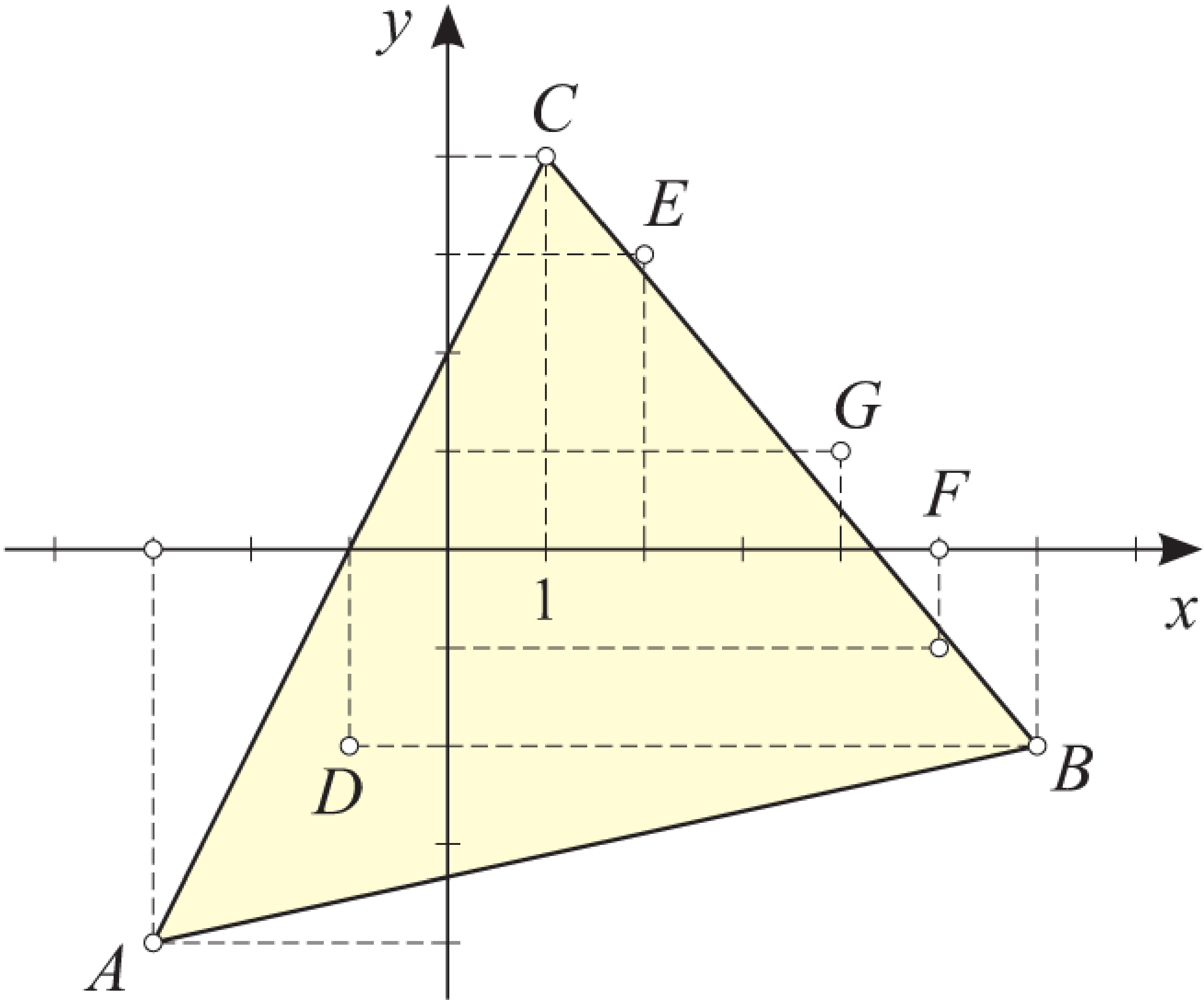
$$-8y-32=0$$
$$y=-4$$
$$2x+4+2=0$$
$$2x=-6 \qquad\qquad A(-3,-4)$$
$$x=-3$$

$$a \cap c \qquad \left.\begin{array}{rl} 2x-9y-30=0 & / \cdot (-3) \\ 6x+5y-26=0 & \end{array}\right\} -$$

$$32y+64=0$$
$$y=-2$$
$$2x-9\cdot(-2)-30=0$$
$$2x=12 \qquad\qquad B(6,-2)$$
$$x=6$$

$$b \cap c \qquad \left.\begin{array}{rl} 2x-y+2=0 & / \cdot (-3) \\ 6x+5y-26=0 & \end{array}\right\} -$$

$$8y-32=0$$
$$y=4$$
$$2x-4+2=0$$
$$2x=2 \qquad\qquad C(1,4)$$
$$x=1$$



y
C
E
G
F
1
x
D
B
A

ZADATAK 8.1.20

Pravci $2x + 3my - 8 = 0$, $mx + y + 3 = 0$ i $3x - y - 5 = 0$ prolaze jednom točkom u ravnini. Koja je to točka?

RJEŠENJE

Iz sustava prvih dviju jednadžbi imao

$$\left.\begin{array}{l} 2x + 3my - 8 = 0 \\ mx + y + 3 = 0 \quad / \cdot (-3m) \end{array}\right\} + \qquad \left.\begin{array}{l} 2x + 3my - 8 = 0 \quad / \cdot m \\ mx + y + 3 = 0 \quad / \cdot (-2) \end{array}\right\} +$$

$$(2 - 3m^2) \cdot x - 8 - 9m = 0 \qquad (3m^2 - 2) \cdot y - 8m - 6 = 0$$

$$\Rightarrow x = \frac{9m + 8}{2 - 3m^2} \qquad \Rightarrow y = \frac{8m + 6}{3m^2 - 2}$$

Uvrstimo li x i y u treću jednadžbu dobit ćemo

$$3 \cdot \frac{9m + 8}{2 - 3m^2} - \frac{8m + 6}{3m^2 - 2} - 5 = 0 \qquad \left(m \neq \frac{2}{3}\right)$$

$$\frac{-27m - 24 - 8m - 6 - 15m^2 + 10}{3m^2 - 2} = 0$$

$$-15m^2 - 35m - 20 = 0 \quad / : (-5)$$

$$3m^2 + 7m + 4 = 0$$

$$m_{1,2} = \frac{-7 \pm \sqrt{49 - 48}}{6} = \frac{-7 \pm 1}{6}$$

$$m_1 = \frac{-7 + 1}{6} = -1, \quad m_2 = \frac{-7 - 1}{6} = -\frac{4}{3}.$$

Za $m = -1$ dobijemo pravce

$$\begin{array}{l} 2x - 3y - 8 = 0 \\ -x + y + 3 = 0 \Rightarrow y = x - 3 \\ 3x - y - 5 = 0 \\ \hline 2x - 3(x - 3) - 8 = 0 \\ -x + 1 = 0 \Rightarrow x = 1, \quad y = -2. \end{array}$$

Tri se pravca sijeku u točki $(1, -2)$. Za $m = -\frac{4}{3}$ dobijemo pravce

$$\begin{array}{l} 2x - 4y - 8 = 0 \\ -\frac{4}{3}x + y + 3 = 0 \\ 3x - y - 5 = 0 \Rightarrow y = 3x - 5 \\ \hline 2x - 4(3x - 5) - 8 = 0 \\ -10x + 12 = 0 \Rightarrow x = \frac{6}{5}, \quad y = -\frac{7}{5}. \end{array}$$

Tri se pravca sijeku u točki $\left(\frac{6}{5}, -\frac{7}{5}\right)$.

ZADATAK 8.1.21

Odredi realni broj a tako da sjecište pravaca $ax + 2y - 1 = 0$ i $2x + ay + 3 = 0$ pripada pravcu $x - y = 3$.

RJEŠENJE

$$\begin{aligned} p \; &\ldots \; ax + 2y - 1 = 0 \\ q \; &\ldots \; 2x + ay + 3 = 0 \\ r \; &\ldots \; y = x - 3 \;\Rightarrow\; T(x_0, x_0 - 3) \end{aligned}$$

$$a \cap b \qquad \begin{aligned} & ax_0 + 2(x_0 - 3) = 0 \\ & 2x_0 + a(x_0 - 3) + 3 = 0 \end{aligned}$$

$$\begin{aligned} & ax_0 + 2x_0 - 6 = 0 \\ & 2x_0 + ax_0 - 3a + 3 = 0 \end{aligned}$$

Oduzimanjem jednakosti dobije se: $-6 + 3a - 1 - 3 = 0, \quad a = \frac{10}{3}$ Uvrstimo dobivenu vrijednost za a u jednadžbe pravaca p i q:

$$\begin{aligned} p \; &\ldots \; \frac{10}{3} \cdot x + 2y - 1 = 0 \quad / \cdot 3 \\ q \; &\ldots \; 2x + \frac{10}{3} \cdot y + 3 = 0 \quad / \cdot 3 \end{aligned}$$

$$\begin{aligned} & 10x + 6y - 3 = 0 \quad / \cdot 6 \\ & 6x + 10y + 9 = 0 \quad / \cdot (-10) \end{aligned}$$

Zbrajanjem dobijemo:
$$-64y - 108 = 0$$
$$-64y = 108$$
$$y = -\frac{27}{16}$$

$$10x + 6 \cdot \left(-\frac{27}{16}\right) - 3 = 0$$

$$10x = 3 + \frac{81}{8}$$

$$x = \frac{21}{16} \quad \Rightarrow \quad T\left(\frac{21}{16}, -\frac{27}{16}\right)$$

ZADATAK 8.1.22

Odredi koeficijent c tako da pravac $x + 4y + c = 0$ prolazi sjecištem pravaca $3x - 2y = 0$ i $3x - 4y + 12 = 0$.

RJEŠENJE

$$p \ \dots \ x + 4y + c = 0$$
$$q \ \dots \ 3x - 2y = 0$$
$$r \ \dots \ 3x - 4y + 12 = 0$$

$q \cap r$

$$3x - 2y = 0$$
$$3x - 4y + 12 = 0$$

Oduzimanjem jednakosti:

$$2y - 12 = 0$$
$$y = 6$$
$$3x - 2 \cdot 6 = 0$$
$$3x = 12$$
$$x = 4 \quad \Longrightarrow \quad T(4, 6)$$
$$T(4, 6) \in p \ \Longrightarrow \ 4 + 4 \cdot 6 + c = 0, \quad c = -28$$

ZADATAK 8.1.23

Kolika je površina trokuta što ga s osi apscisa zatvaraju pravci $3x - 4y = 0$ i $3x - 2y - 6 = 0$?

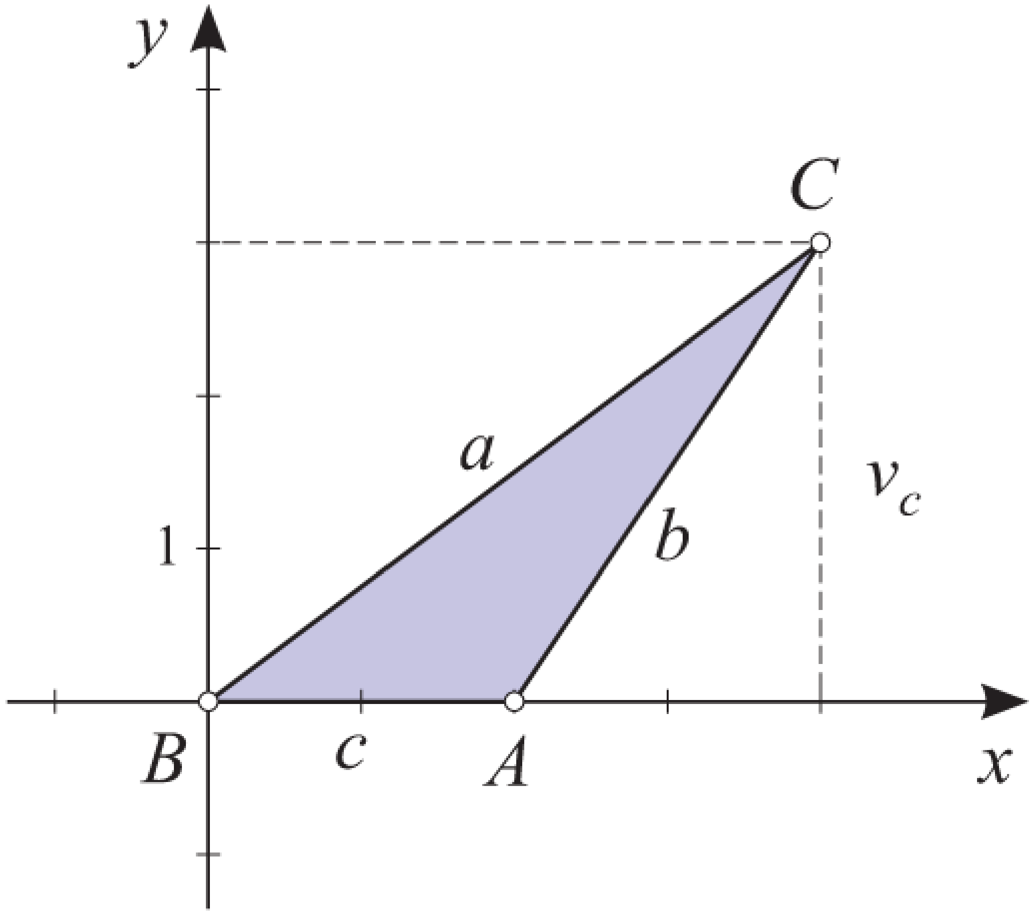
RJEŠENJE

Neka su A, B i C vrhovi traženog trokuta.

$$a \; \ldots \; 3x - 4y = 0$$
$$b \; \ldots \; 3x - 2y - 6 = 0$$
$$c \; \ldots \; y = 0 \;\; \text{(os apscisa)}$$

$$A \in b \cap c \quad 3x - 6 = 0$$
$$x = 2 \;\Rightarrow\; A(2, 0)$$
$$B \in a \cap c \quad 3x = 0$$
$$x = 0 \;\Rightarrow\; B(0, 0)$$
$$C \in a \cap b \quad 3x - 4y = 0$$
$$3x - 2y - 6 = 0$$

$$\text{Oduzimanjem dobijemo:} \quad -2y + 6 = 0$$
$$y = 3$$
$$3x - 4 \cdot 3 = 0$$
$$x = 4 \;\Rightarrow\; C(4, 3)$$

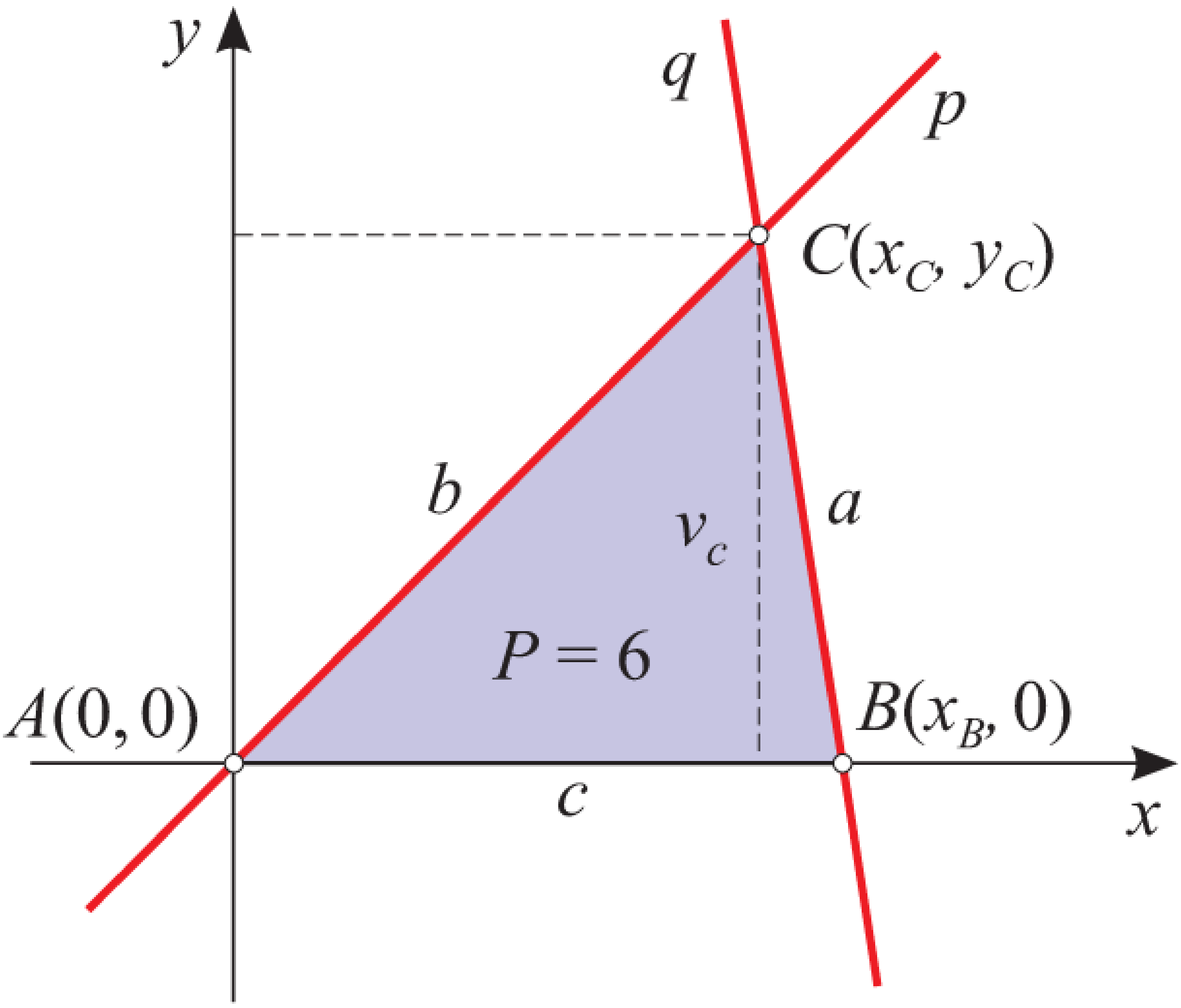


$$P = \frac{c \cdot v_c}{2} = \frac{2 \cdot 3}{2} = 3.$$

ZADATAK 8.1.24

Ishodištem koordinatnog sustava položi pravac koji će s pravcem $3x + 2y - 12 = 0$ i osi ordinata tvoriti trokut površine 6.

RJEŠENJE



$$p \ \ldots \ 3x + 2y - 12 = 0$$

$$q \ \ldots \ y = kx \quad \text{prolazi ishodištem}$$

$$P = 6 = \frac{c \cdot v_a}{2}$$

$c = x_B$, gdje je $B(x_B, 0)$ sjecište pravca p s osi ordinata.

$$3x_B + 2 \cdot 0 - 12 = 0 \ \Rightarrow \ c = x_B = 4 \ \Rightarrow \ B(4, 0)$$

$$P = \frac{c \cdot v_C}{2}$$

$$6 = \frac{4 \cdot v_C}{2} \quad / \cdot 2$$

$$v_C = 3$$

v_C je apsolutna vrijednost ordinate točke $C \ldots p \cap q \ \Rightarrow \ C(x_C, \pm 3)$

1) $y_C = -3$

$3x_C + 2 \cdot (-3) - 12 = 0$

$3x_C = 18$

$x_C = 6$

$C_1(6, 3)$

$-3 = k \cdot 6$

$k = -\frac{1}{2}$

$q_1 \ \dots \ y = -\frac{1}{2}x$

2) $y_C = 3$

$3x_C + 2 \cdot 3 - 12 = 0$

$3x_C = 6$

$x_C = 2$

$C_2(2, 3)$

$3 = k \cdot 2$

$k = \frac{3}{2}$

$q_2 \ \dots \ y = \frac{3}{2}x$

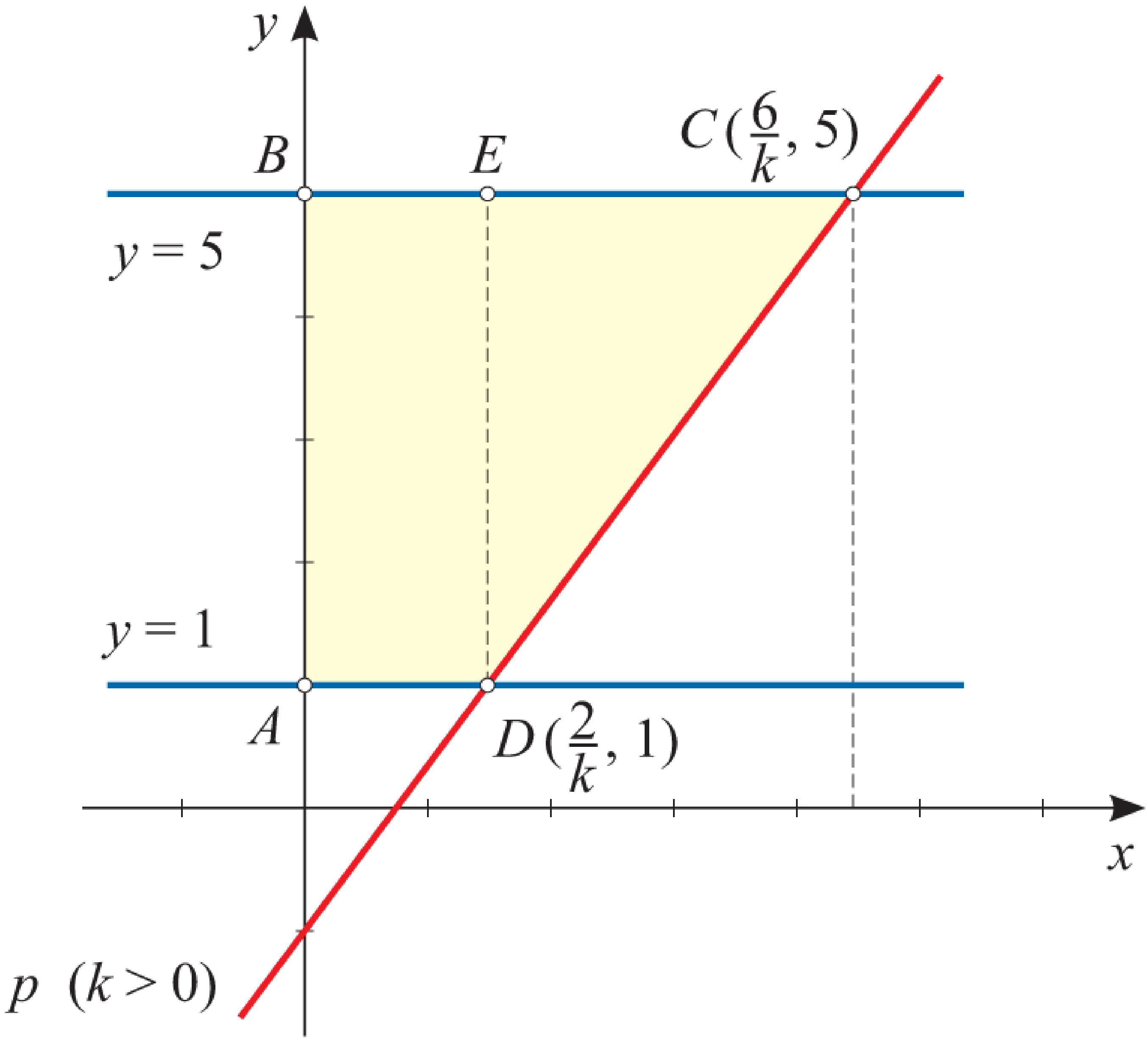
ZADATAK 8.1.25

Odredi koeficijent $k > 0$ tako da pravci $y = kx - 1$, $y = 1$ i $y = 5$ s osi ordinata zatvaraju konveksni četverokut površine 12.

RJEŠENJE

$a \;\ldots\; y = 5$ $b \;\ldots\; y = 1$ $p \;\ldots\; y = kx - 1 \;\Rightarrow\; x = \dfrac{y+1}{k}$ $C = p \cap a = \left(\dfrac{5+1}{k}, 5\right) = \left(\dfrac{6}{k}, 5\right)$

$D = p \cap b = \left(\dfrac{1+1}{k}, 1\right) = \left(\dfrac{2}{k}, 1\right)$



$k > 0$

$P = P_{ABED} + P_{ECD}$

$$12 = 4 \cdot \frac{2}{k} + \frac{4 \cdot \left(\frac{6}{k} - \frac{2}{k}\right)}{2}$$

$$12 = \frac{8}{k} + \frac{8}{k}$$

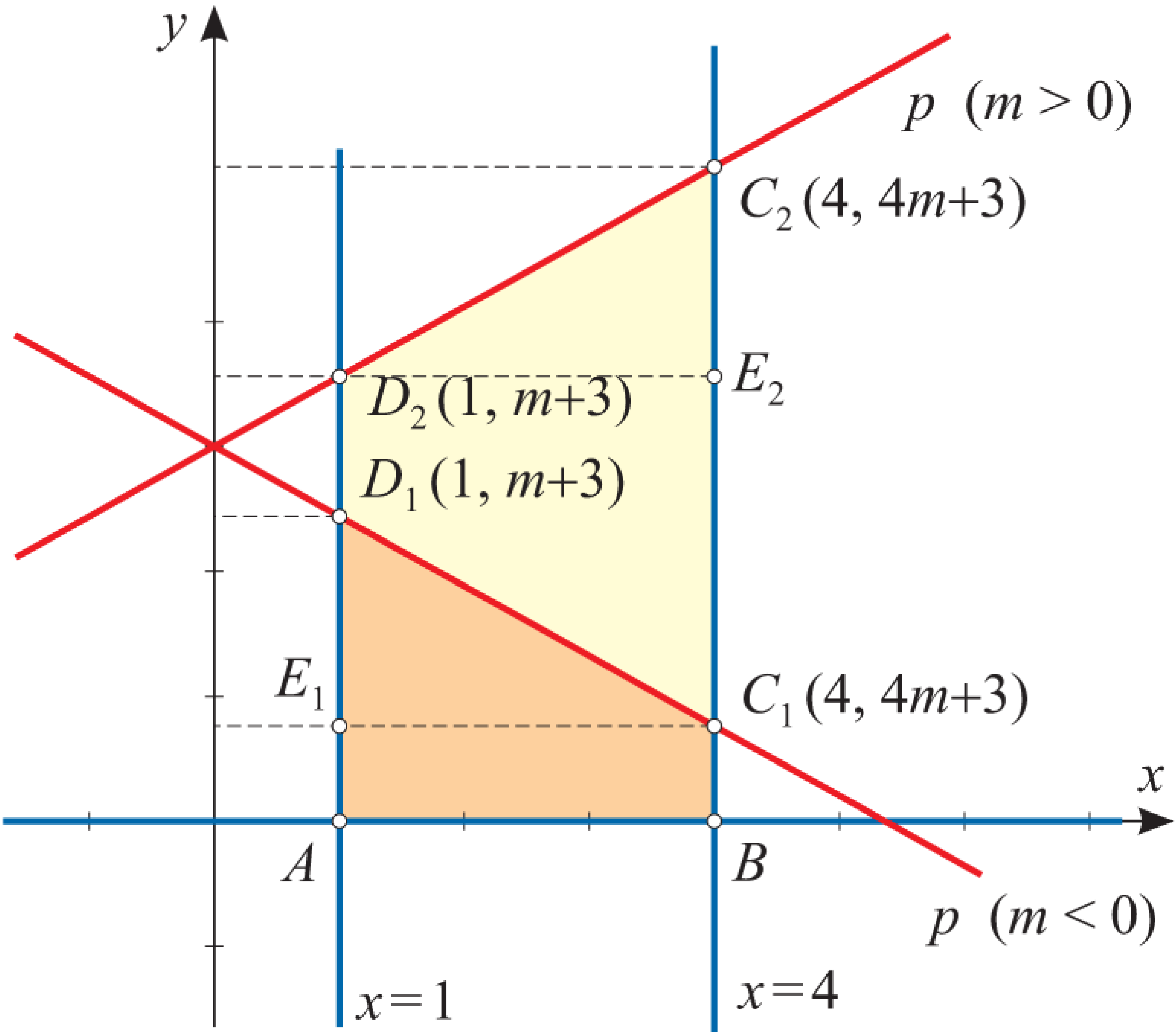
$$12 = \frac{16}{k}$$

$$k = \frac{3}{4}$$

ZADATAK 8.1.26

Za koje je m površina konveksnog četverokuta što ga zatvaraju pravci $y = mx + 3$, $x = 1$, $x = 4$ s osi apscisa jednaka 12?

RJEŠENJE



$m > 0$

$P = P_{ABE_2D_2} + P_{E_2C_2D_2}$

$$12 = 3(m+3) + \frac{3[(4m+3) - (m+3)]}{2}$$

$$12 = 3m + 9 + \frac{3}{2} \cdot 3m$$

$$12 = 3m + 9 + \frac{9}{2}m$$

$$\frac{15m}{2} = 3 \quad / \cdot \frac{2}{15}$$

$$m = \frac{2}{5}$$

$m < 0$

$P = P_{ABC_1E_1} + P_{E_1C_1D_1}$

$$12 = 3(4m+3) + \frac{3}{2}(m + 3 - 4m - 3)$$

$$3 = 12m - \frac{9}{2}m$$

$$\frac{15m}{2} = 3 \quad / \cdot \frac{2}{15}$$

$$m = \frac{2}{5}$$

nije rješenje jer je $m < 0$

ZADATAK 8.2.1

Jednadžba pravca dana je u implicitnom obliku. Prevedi je u segmentni oblik. Nacrtaj potom pravac.

1) $x - 4y + 4 = 0$

2) $3x + 2y - 6 = 0$

3) $4x - 3y - 12 = 0$

4) $2x + y - 2 = 0$

5) $x + y + 5 = 0$

6) $x + 3y - 9 = 0$.

RJEŠENJE

1)

$$x - 4y + 4 = 0$$
$$x - 4y = -4$$
$$x - 4y = -4 \quad / : (-4)$$
$$\frac{x}{-4} + \frac{y}{1} = 1$$

2)

$$3x + 2y - 6 = 0$$
$$3x + 2y = 6$$
$$3x + 2y = 6 \quad / : 6$$
$$\frac{x}{2} + \frac{y}{3} = 1$$

3)

$$4x - 3y - 12 = 0$$
$$4x - 3y = 12 \quad / : 12$$
$$\frac{x}{3} + \frac{y}{-4} = 1$$

4)

$$2x + y - 2 = 0$$
$$2x + y = 2 \quad / : 2$$
$$\frac{x}{1} + \frac{y}{2} = 1$$

5)

$$x + y + 5 = 0$$
$$x + y = -5 \quad / : (-5)$$
$$\frac{x}{-5} + \frac{y}{-5} = 1$$

6)

$$x + 3y - 9 = 0$$
$$x + 3y = 9 \quad / : 9$$
$$\frac{x}{9} + \frac{y}{3} = 1$$

ZADATAK 8.2.2

Jednadžbu pravca prevedi iz eksplicitnog u segmentni oblik i nacrtaj pravac:

1) $y = x + 1$

2) $y = 2x - 4$

3) $y = -\frac{1}{2}x + 2$

4) $y = \frac{2}{3}x - 4$

5) $y = 3x + 6$

6) $y = -\frac{1}{3}x + \frac{4}{3}$.

RJEŠENJE

1)

$$y = x + 1$$
$$-x + y = 1$$
$$\frac{x}{-1} + \frac{y}{1} = 1$$

2)

$$y = 2x - 4$$
$$2x - y = 4 \quad /:4$$
$$\frac{x}{2} + \frac{y}{-4} = 1$$

3)

$$y = -\frac{1}{2}x + 2$$
$$\frac{1}{2}x + y = 2 \quad /:2$$
$$\frac{x}{4} + \frac{y}{2} = 1$$

4)

$$y = \frac{2}{3}x - 4$$
$$\frac{2}{3}x - y = 4 \quad /:4$$
$$\frac{x}{6} + \frac{y}{-4} = 1$$

5)

$$y = 3x + 6$$
$$-3x + y = 6 \quad /:6$$
$$\frac{x}{-2} + \frac{y}{6} = 1$$

6)

$$y = -\frac{1}{3}x + \frac{4}{3}$$

$$\frac{1}{3}x + y = \frac{4}{3} \quad \Big/ : \frac{4}{3}$$

$$\frac{x}{4} + \frac{y}{\frac{4}{3}} = 1$$

ZADATAK 8.2.3

Odredi odsječke danog pravca na koordinatnim osima:

1) $x + 3y + 9 = 0$

2) $3x - 2y - 6 = 0$

3) $4x + 6y + 8 = 0$

4) $5x - 3y + 11 = 0.$

RJEŠENJE

1)

$$x + 3y + 9 = 0$$
$$x + 3y = -9 \quad / : (-9)$$
$$\frac{x}{-9} + \frac{y}{-3} = 1$$

Odsječak na osi x: $m = -9$

Odsječak na osi y: $n = -3$

2)

$$3x - 2y - 6 = 0$$
$$3x - 2y = 6 \quad / : 6$$
$$\frac{x}{2} + \frac{y}{-3} = 1 \implies m = 2, \quad n = -3$$

3)

$$4x + 6y + 8 = 0$$
$$4x + 6y = -8 \quad / : (-8)$$
$$\frac{x}{-2} + \frac{3y}{-4} = 1$$
$$\frac{x}{-2} + \frac{y}{-\frac{4}{3}} = 1 \implies m = -2, \quad n = -\frac{4}{3}$$

4)

$$5x - 3y + 11 = 0$$
$$5x - 3y = -11 \quad / : (-11)$$
$$\frac{5x}{-11} + \frac{-3y}{-11} = 1$$
$$\frac{x}{-\frac{11}{5}} + \frac{y}{\frac{11}{3}} = 1 \implies m = -\frac{11}{5}, \quad n = \frac{11}{3}$$

ZADATAK 8.2.4
Kolika je duljina odsječka što ga koordinatne osi odsijecaju na pravcu:

1) $4x - 3y + 12 = 0$

2) $\frac{2}{3}x - \frac{1}{2}y = 1$

3) $y = \frac{1}{2}x + 4$

4) $y = -\frac{3}{4}x - \frac{5}{4}$?

RJEŠENJE
1)

$$4x - 3y + 12 = 0$$
$$4x - 3y = -12 \quad / : (-12)$$
$$\frac{4x}{-12} + \frac{-3y}{-12} = 1$$
$$\frac{x}{-3} + \frac{y}{4} = 1 \Rightarrow m = -3, \quad n = 4$$
$$d = \sqrt{m^2 + n^2} = \sqrt{9 + 16} = 5, \qquad d = 5$$

2)

$$\frac{2}{3}x - \frac{1}{2}y = 1$$
$$\frac{x}{\frac{3}{2}} + \frac{y}{-2} = 1 \Rightarrow m = \frac{3}{2}, \quad n = -2$$
$$d = \sqrt{m^2 + n^2} = \sqrt{\frac{9}{4} + 4} = \sqrt{\frac{25}{4}} = \frac{5}{2}, \qquad d = \frac{5}{2}$$

3)

$$y = \frac{1}{2}x + 4$$
$$y - \frac{1}{2}x = 4 \quad / : 4$$
$$\frac{x}{-8} + \frac{y}{4} = 1 \Rightarrow m = -8, \quad n = 4$$
$$d = \sqrt{m^2 + n^2} = \sqrt{64 + 16} = \sqrt{80}, \qquad d = 4\sqrt{5}$$

4)

$$y = -\frac{3}{4}x - \frac{5}{4}$$
$$\frac{3}{4}x + y = -\frac{5}{4} \quad / : \left(-\frac{5}{4}\right)$$
$$\frac{x}{-\frac{5}{3}} + \frac{y}{-\frac{5}{4}} = 1 \Rightarrow m = -\frac{5}{3}, \quad n = -\frac{5}{4}$$
$$d = \sqrt{m^2 + n^2} = \sqrt{\frac{25}{9} + \frac{25}{16}} = \sqrt{\frac{625}{144}}, \qquad d = \frac{25}{12}$$

ZADATAK 8.2.5

Kolika je površina trokuta što ga s koordinatnim osima zatvara pravac čija jednadžba glasi:

1) $\frac{x}{11} + \frac{y}{-12} = 1$

2) $\frac{x}{-3} + y = 1$

3) $\frac{3}{4}x - \frac{5}{7}y = 1$

4) $2x - \frac{1}{4}y = 1$?

RJEŠENJE

1)

$$\frac{x}{11} + \frac{y}{-12} = 1 \;\Rightarrow\; m = -11, \quad n = -12$$

$$P = \frac{|m \cdot n|}{2} = \frac{11 \cdot 12}{2} = 66, \qquad P = 66$$

2)

$$\frac{x}{-3} + y = 1 \;\Rightarrow\; m = -3, \quad n = 1$$

$$P = \frac{|m \cdot n|}{2} = \frac{3 \cdot 1}{2} = \frac{3}{2}, \qquad P = \frac{3}{2}$$

3)

$$\frac{3}{4}x - \frac{5}{7}y = 1$$

$$\frac{x}{\frac{4}{3}} + \frac{y}{-\frac{7}{5}} = 1 \;\Rightarrow\; m = \frac{4}{3}, \quad n = \frac{7}{5}$$

$$P = \frac{|m \cdot n|}{2} = \frac{\frac{4}{3} \cdot \frac{7}{5}}{2} = \frac{14}{15}, \qquad P = \frac{14}{15}$$

4)

$$2x - \frac{1}{4}y = 1$$

$$\frac{x}{\frac{1}{2}} + \frac{y}{-4} = 1 \;\Rightarrow\; m = \frac{1}{2}, \quad n = -4$$

$$P = \frac{|m \cdot n|}{2} = \frac{\frac{1}{2} \cdot 4}{2} = 1, \qquad P = 1$$

ZADATAK 8.2.6

Kolika je površina trokuta što ga s koordinatnim osima zatvara pravac:

1) $2x - 5y + 10 = 0$

2) $x + 6y + 6 = 0$

3) $3x + y - 3 = 0$

4) $y = \frac{1}{2}x - \frac{5}{2}$

5) $y = -\frac{3}{2}x - 5.$

RJEŠENJE

1)

$$2x - 5y + 10 = 0$$
$$2x - 5y = -10 \quad / : (-10)$$
$$\frac{x}{-5} + \frac{y}{2} = 1 \Rightarrow m = -5, \quad n = 2$$
$$P = \frac{|m \cdot n|}{2} = \frac{5 \cdot 2}{2} = 5, \qquad P = 5$$

2)

$$x + 6y + 6 = 0$$
$$x + 6y = -6 \quad / : (-6)$$
$$\frac{x}{-6} + \frac{y}{-1} = 1 \Rightarrow m = -6, \quad n = -1$$
$$P = \frac{|m \cdot n|}{2} = \frac{6 \cdot 1}{2} = 3, \qquad P = 3$$

3)

$$3x + y - 3 = 0$$
$$3x + y = 3 \quad / : 3$$
$$\frac{x}{1} + \frac{y}{3} = 1 \Rightarrow m = 1, \quad n = 3$$
$$P = \frac{|m \cdot n|}{2} = \frac{1 \cdot 3}{2} = 1.5, \qquad P = 1.5$$

4)

$$y = \frac{1}{2}x - \frac{5}{2}$$
$$\frac{1}{2}x - y = \frac{5}{2} \quad / : \frac{5}{2}$$
$$\frac{x}{5} + \frac{y}{-\frac{5}{2}} = 1 \Rightarrow m = 5, \quad n = -\frac{5}{2}$$
$$P = \frac{|m \cdot n|}{2} = \frac{5 \cdot \frac{5}{2}}{2} = \frac{25}{4}, \qquad P = \frac{25}{4}$$

5)

$$y = -\frac{3}{2}x - 5$$

$$\frac{3}{2}x + y = -5 \quad / : (-5)$$

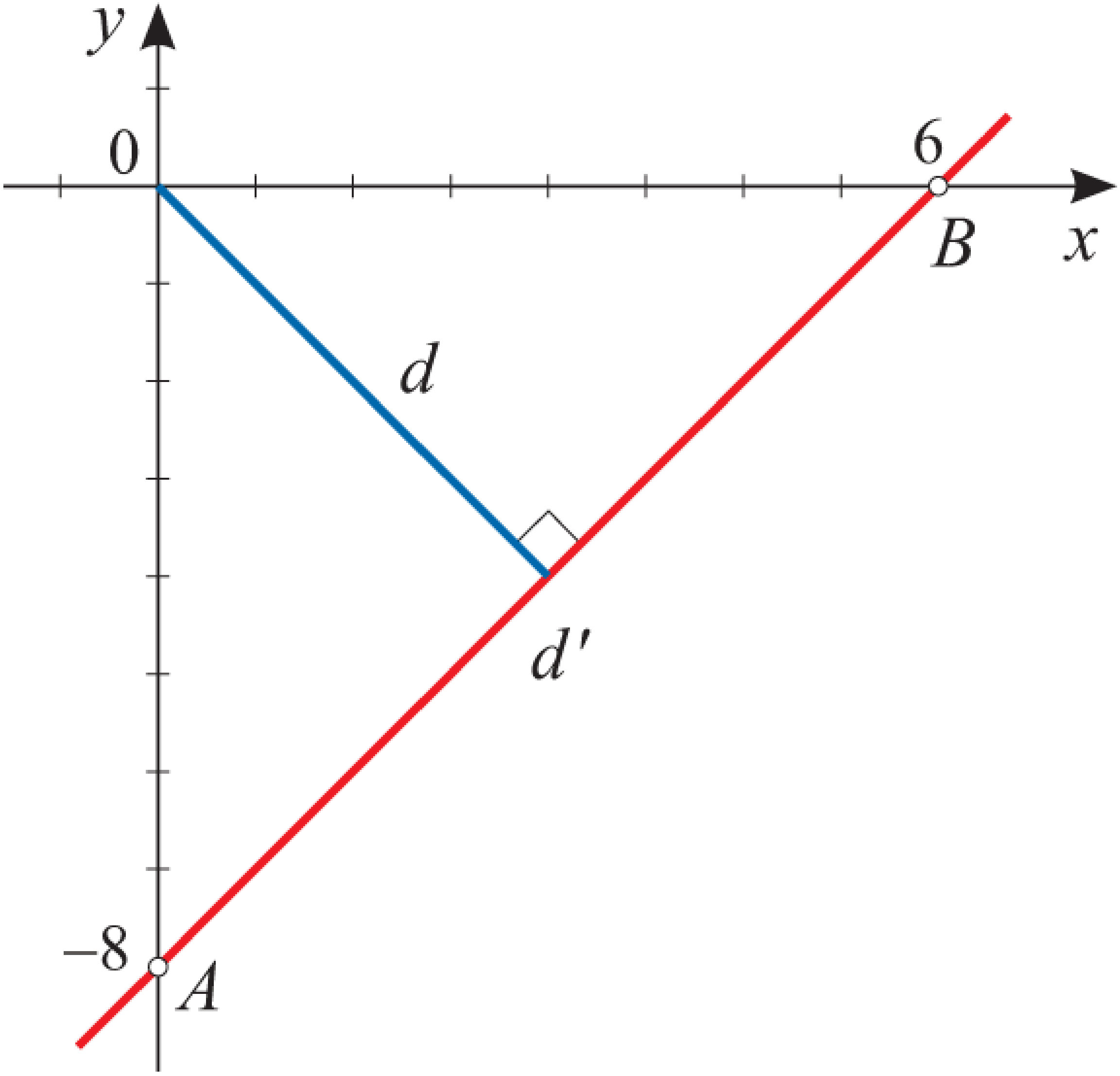
$$\frac{x}{-\frac{10}{3}} + \frac{y}{-5} = 1 \implies m = -\frac{10}{3}, \quad n = -5$$

$$P = \frac{|m \cdot n|}{2} = \frac{\frac{10}{3} \cdot 5}{2} = \frac{25}{3}, \qquad P = \frac{25}{3}$$

ZADATAK 8.2.7

Kolika je udaljenost pravca $\frac{x}{6} - \frac{y}{8} = 1$ od ishodišta koordinatnog sustava?

RJEŠENJE



$$\frac{x}{6} - \frac{y}{8} = 1$$

$$\frac{x}{6} + \frac{y}{-8} = 1 \implies m = 6, \quad n = -8$$

$$P = \frac{|m \cdot n|}{2} = \frac{6 \cdot 8}{2} = 24$$

$$d' = \sqrt{m^2 + n^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

$$P = \frac{d' \cdot d}{2} \implies d = \frac{2P}{d} = \frac{2 \cdot 24}{10} = 4.8$$

ZADATAK 8.2.8

Kolika je udaljenost pravca $\frac{x}{3} + \frac{y}{-6} = 1$ od ishodišta koordinatnog sustava?

RJEŠENJE

$$\frac{x}{3} + \frac{y}{-6} = 1 \implies m = 3, \quad n = -6$$

$$P = \frac{|m \cdot n|}{2} = \frac{3 \cdot 6}{2} = 9$$

$$d' = \sqrt{m^2 + n^2} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$$

$$d = \frac{2P}{d'} = \frac{2 \cdot 9}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$$

ZADATAK 8.2.9

Odredi realni broj m tako da duljina odsječka pravca $2x + my + 2m = 0$ između koordinatnih osi bude jednaka $\frac{5}{2}$.

RJEŠENJE

$$2x + my + 2m = 0$$

$$2x + my = -2m \quad / : (-2m)$$

$$\frac{x}{-m} + \frac{y}{-2} = 1$$

$$d = \sqrt{m^2 + n^2}$$

$$\frac{5}{2} = \sqrt{(-m)^2 + (-2)^2}$$

$$\frac{5}{2} = \sqrt{m^2 + 4} \quad /^2$$

$$\frac{25}{4} = m^2 + 4$$

$$m^2 = \frac{25}{4} - 4$$

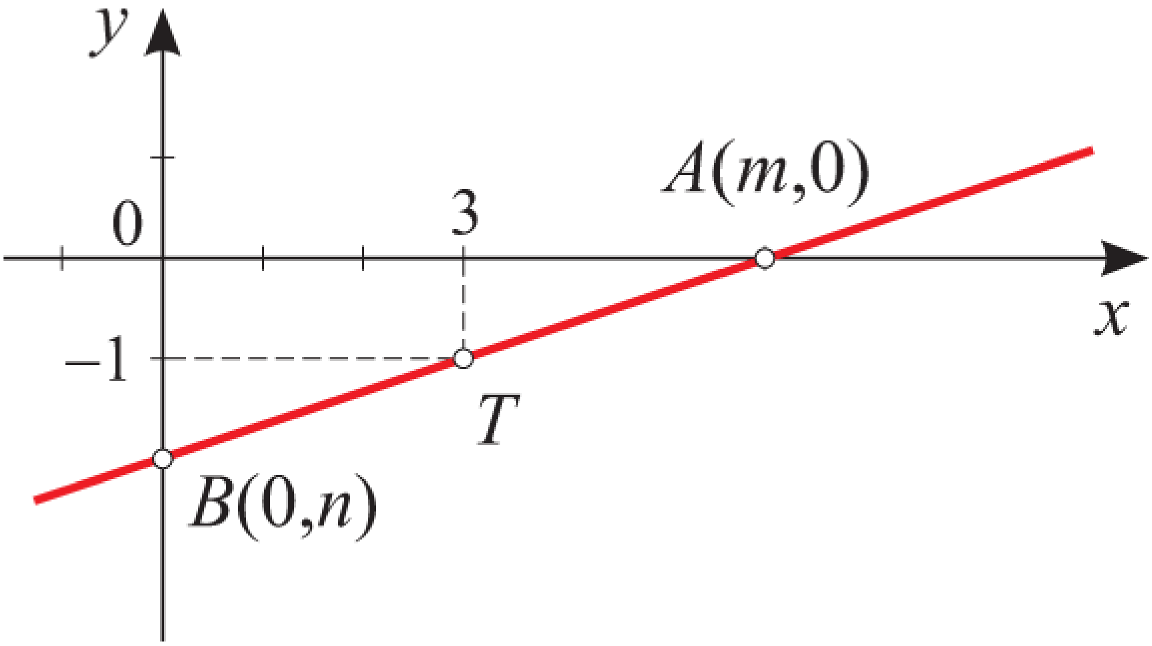
$$m^2 = \frac{9}{4}$$

$$m = \pm\frac{3}{2}$$

ZADATAK 8.2.10

Točkom $T(3, -1)$ položi pravac tako da T bude polovište odsječka što ga na pravcu odsijecaju koordinatne osi.

RJEŠENJE



$$x_T = \frac{m+0}{2} \Rightarrow 3 = \frac{m+0}{2}$$

$$y_T = \frac{0+n}{2} \Rightarrow -1 = \frac{0+n}{2}$$

$$m = 6$$

$$n = -2$$

$$\frac{x}{6} + \frac{y}{-2} = 1$$

ZADATAK 8.2.11

Točkom $T(-4, -1)$ položi pravac tako da ona u omjeru $\lambda = 2$ dijeli odsječak tog pravca između koordinatnih osi.

RJEŠENJE

$T(-4, -1), \quad \lambda = 2$

$$1^\circ \quad x_T = \frac{m + \lambda \cdot 0}{1+\lambda} \Rightarrow -4 = \frac{m}{1+2} \quad / \cdot 3$$

$$y_T = \frac{0 + \lambda \cdot n}{1+\lambda} \Rightarrow -1 = \frac{2n}{1+2} \quad / \cdot 3$$

$$m = -12$$

$$2n = -3 \Rightarrow n = -\frac{3}{2}$$

$$\frac{x}{-12} + \frac{y}{-\frac{3}{2}} = 1$$

$$-\frac{x}{12} - \frac{2y}{3} = 1 \quad / \cdot (-12)$$

$$x + 8y = -12$$

$$x + 8y + 12 = 0$$

$$2^\circ \quad x_T = \frac{0 + \lambda \cdot m}{1+\lambda} \Rightarrow -4 = \frac{2m}{1+2} \quad / \cdot 3$$

$$y_T = \frac{n + \lambda \cdot 0}{1+\lambda} \Rightarrow -1 = \frac{n+0}{1+2} \quad / \cdot 3$$

$$2m = -12 \Rightarrow m = -6$$

$$n = -3$$

$$\frac{x}{-6} + \frac{y}{-3} = 1 \quad / \cdot (-6)$$

$$x + 2y + 6 = 0$$

ZADATAK 8.2.12

Odredi jednadžbu pravca koji prolazi točkom $T(5, 1)$ i na koordinatnim osima odsijeca odsječke jednakih duljina.

RJEŠENJE

$T(5, 1)$

$m = n$	$m = -n$
$\frac{x}{m} + \frac{y}{n} = 1$	$\frac{x}{m} + \frac{y}{n} = 1$
$\frac{5}{m} + \frac{1}{m} = 1 \quad / \cdot m$	$\frac{5}{m} + \frac{1}{-m} = 1 \quad / \cdot m$
$5 + 1 = m$	$5 - 1 = m$
$m = 6$	$m = 4$
$n = 6$	$n = -4$
$\frac{x}{6} + \frac{y}{6} = 1 \quad / \cdot 6$	$\frac{x}{4} + \frac{y}{-4} = 1 \quad / \cdot 4$
$x + y - 6 = 0$	$x - y - 4 = 0$

ZADATAK 8.2.13

Odsječak pravca p na osi ordinata tri puta je veći od njegovog odsječka na osi apscisa. Pravac prolazi točkom $T(3, 3)$. Odredi njegovu jednadžbu.

RJEŠENJE

$T(3, 3)$

$$n = 3m$$
$$\frac{x}{m} + \frac{y}{n} = 1$$
$$\frac{3}{m} + \frac{3}{3m} = 1 \quad / \cdot 3m$$
$$9 + 3 = 3m$$
$$m = 4$$
$$n = 12$$
$$\frac{x}{4} + \frac{y}{12} = 1$$

$$n = -3m$$
$$\frac{x}{m} + \frac{y}{n} = 1$$
$$\frac{3}{m} + \frac{3}{-3m} = 1 \quad / \cdot (-3m)$$
$$-9 + 3 = -3m$$
$$m = 2$$
$$n = -6$$
$$\frac{x}{2} + \frac{y}{-6} = 1$$

ZADATAK 8.2.14

Odredi jednadžbu pravca koji prolazi točkom $T(8, 6)$, a s koordinatnim osima zatvara trokut površine 12.

RJEŠENJE

$T(8, 6),\ P = 12P = \dfrac{|m \cdot n|}{2}, \quad 12 = \dfrac{|m \cdot n|}{2}, \quad |m| = \dfrac{24}{|n|}$

$1°\quad m = \dfrac{24}{n}$

$$\frac{x}{m} + \frac{y}{n} = 1$$

$$\frac{8}{\frac{24}{n}} + \frac{6}{n} = 1$$

$$\frac{8n}{24} + \frac{6}{n} = 1 \quad / \cdot 24n$$

$$8n^2 + 144 = 24n$$

$$8n^2 - 24n + 144 = 0$$

$$n^2 - 3n + 18 = 0$$

$$n_{1,2} = \frac{3 \pm \sqrt{9 - 72}}{2} \quad \text{(nema rješenja)}$$

$2°\quad m = -\dfrac{24}{n}$

$$\frac{x}{m} + \frac{y}{n} = 1$$

$$\frac{8}{-\frac{24}{n}} + \frac{6}{n} = 1$$

$$-\frac{8n}{24} + \frac{6}{n} = 1 \quad / \cdot (-24n)$$

$$8n^2 - 144 = -24n$$

$$8n^2 + 24n - 144 = 0$$

$$n^2 + 3n - 18 = 0$$

$$n_{1,2} = \frac{-3 \pm \sqrt{9 + 72}}{2} = \frac{-3 \pm 9}{2}$$

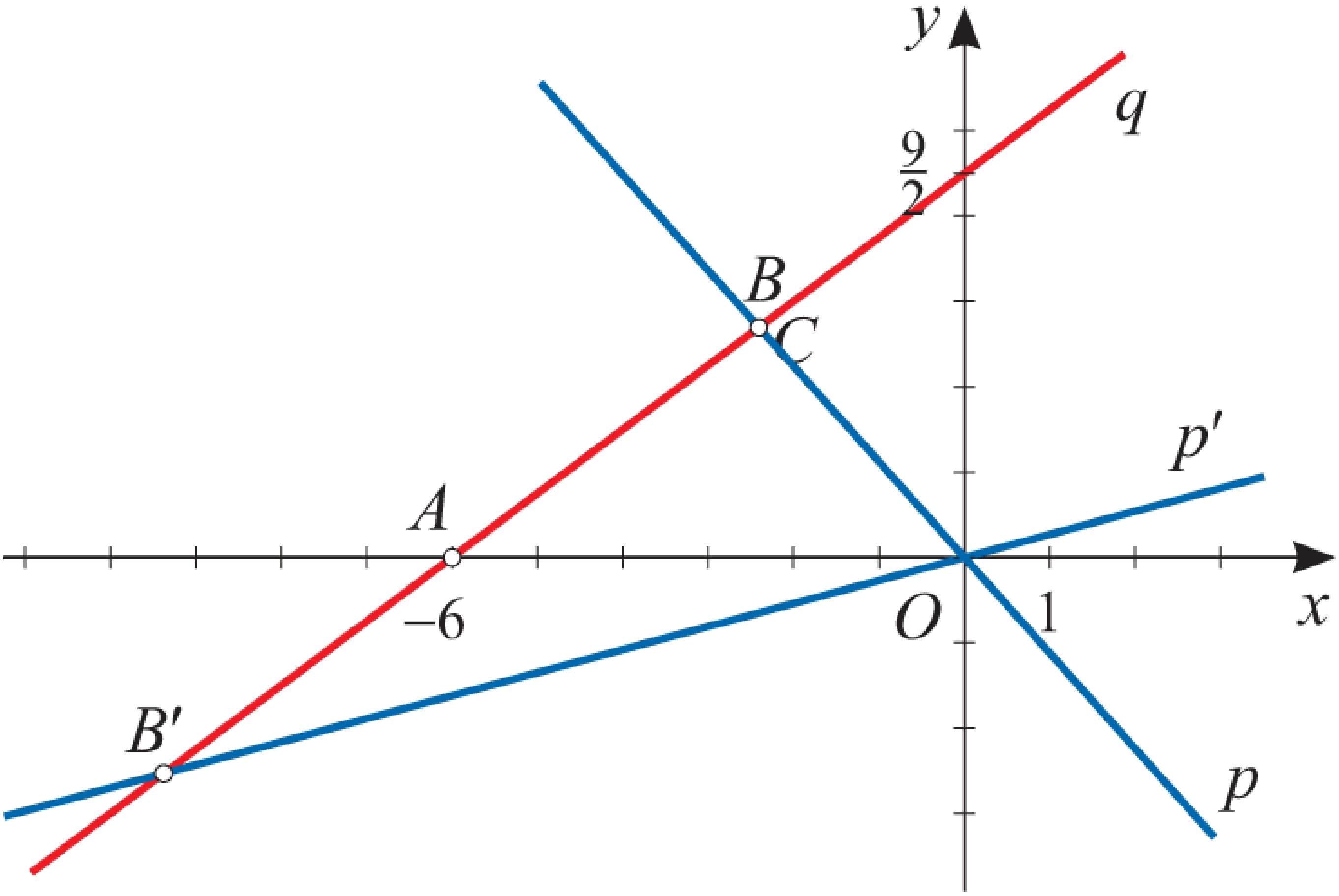
$$n_1 = \frac{-3 - 9}{2} = -6 \implies \frac{x}{4} + \frac{y}{-6} = 1, \quad 3x - 2y - 12 = 0$$

$$n_2 = \frac{-3 + 9}{2} = 3 \implies \frac{x}{-8} + \frac{y}{3} = 1, \quad 3x - 8y + 24 = 0$$

ZADATAK 8.2.15

Ishodištem koordinatnog sustava položi pravac koji će s pravcem $3x - 4y + 18 = 0$ i osi apscisa zatvarati trokut površine 9.

RJEŠENJE



$$q \;\dots\; 3x - 4y + 18 = 0$$

$$\Longrightarrow \; -4y = -3x - 18 \quad / : (-4)$$

$$\Longrightarrow \; y = \frac{3}{4}x + \frac{9}{2}$$

$$p \;\dots\; y = kx$$

$$P = 9 = P_{\Delta AOB} = P_{\Delta AOB'}$$

Koordinate točaka B i B' su (x, kx) pa imamo:

$$kx = \frac{3}{4}x + \frac{9}{2}$$

$$kx - \frac{3}{4}x = \frac{9}{2}$$

$$x\left(k - \frac{3}{4}\right) = \frac{9}{2}$$

$$P_{\Delta} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$\Delta AOB \quad 9 = \frac{1}{2}\left| - 6(0 - kx) + 0 \cdot (kx - 0) + x(0 - 0)\right| \quad / \cdot 2$$

$$\Delta AOB' \quad 9 = \frac{1}{2}\left| - 6(0 - kx) + 0 \cdot (kx - 0) + x(0 - 0)\right|$$

$$18 = |6kx| \quad / : 6$$

$$|kx| = 3$$

$$|k| = \frac{3}{|x|}$$

uvrstimo u (1):

$$x\left(\frac{3}{|x|} - \frac{3}{4}\right) = \frac{9}{2}$$

$$\frac{3x}{|x|} - \frac{3}{4}x = \frac{9}{2} \quad / \cdot 4|x|$$

$$12x - 3x|x| = 18|x|$$

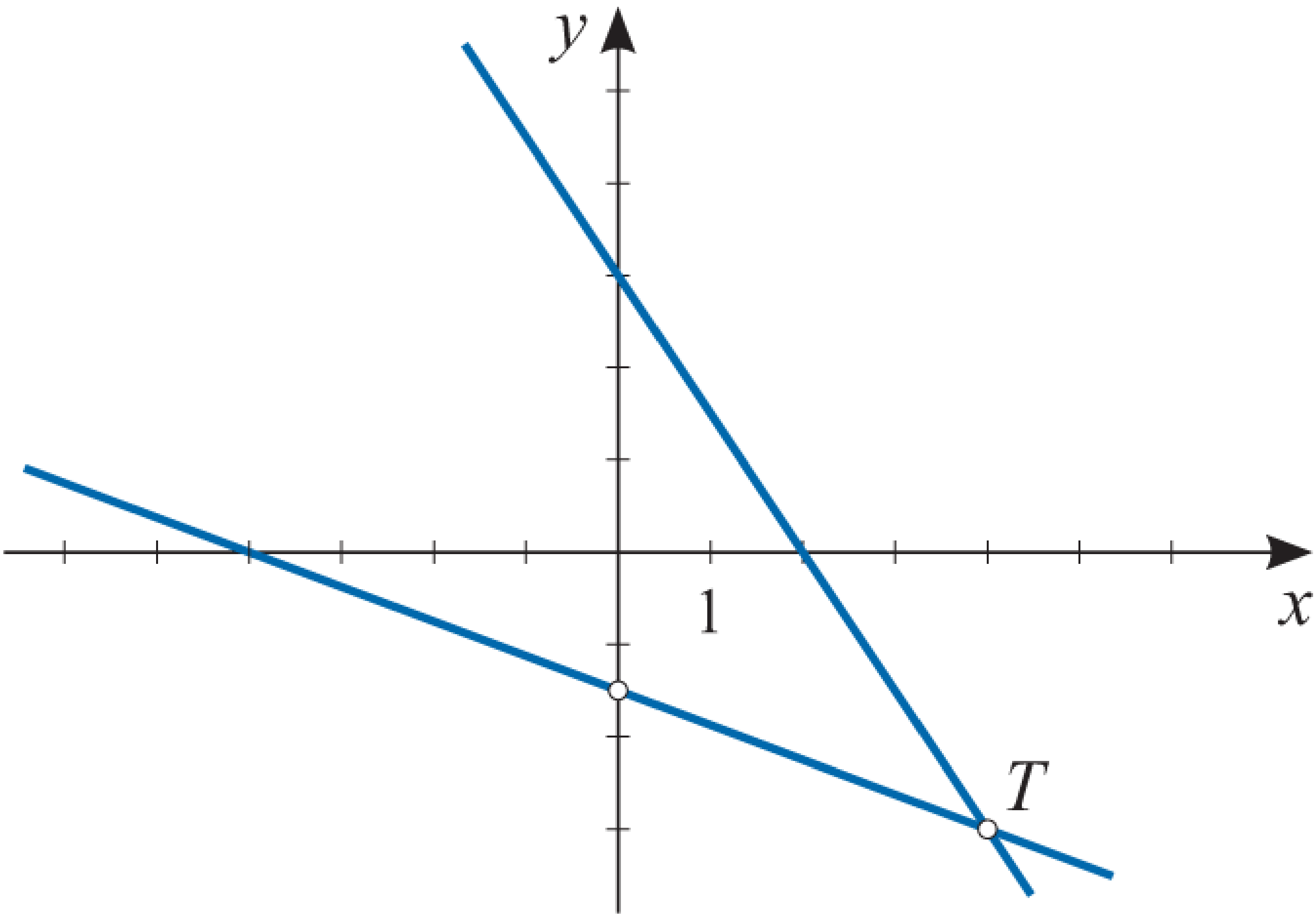
$x < 0$	$x > 0$
$12x + 3x^2 = -18x$	$12x - 3x^2 = 18x$
$3x^2 + 30x = 0$	$-3x^2 - 6x = 0$
$3x(x + 10) = 0$	$-x(x + 2) = 0$
$x = -10$	$x = -2$
$\|k\| = \frac{3}{10} \Rightarrow$ iz slike $\Rightarrow k = \frac{3}{10}$	$\|k\| = \frac{3}{2} \Rightarrow$ iz slike $\Rightarrow k = -\frac{3}{2}$
$y = \frac{3}{10}x$	$y = -\frac{3}{2}x$

ZADATAK 8.2.16

Točkom $T(4,\ -3)$ položi pravac tako da površina trokuta što ga taj pravac tvori s koordinatnim osima bude 3.

RJEŠENJE

$T(4,\ -3)$



$$P = \frac{|m \cdot n|}{2}$$

$$3 = \frac{|m \cdot n|}{2} \quad / \cdot 2$$

$$|m \cdot n| = 6$$

$$|n| = \frac{6}{|m|}$$

$1°\quad n > 0 \implies n = \dfrac{6}{|m|}$

$$\frac{4}{m} + \frac{-3}{\frac{6}{|m|}} = 1$$

$$\frac{4}{m} - \frac{|m|}{2} = 1 \quad / \cdot 2m$$

$$8 - m \cdot |m| = 2m$$

$$m \cdot |m| + 2m - 8 = 0 \qquad (1)$$

a) $\quad m < 0$

$(1) \implies -m^2 + 2m - 8 = 0$

$$m^2 - 2m + 8 = 0$$

$$m_{1,2} = \frac{2 \pm \sqrt{4 - 32}}{2}$$

(nema rješenja)

b) $m > 0$

$(1) \Rightarrow m^2 + 2m - 8 = 0$

$$m_{1,2} = \frac{-2 \pm \sqrt{4 + 32}}{2} = \frac{-2 \pm 6}{2}$$

$$m_1 = \frac{-2 - 6}{2} \text{ (nije rješenje)}$$

$$m_2 = \frac{-2 + 6}{2} = 2 \Rightarrow n = 3$$

$$p \ \dots \ \frac{x}{2} + \frac{y}{3} = 1, \quad 3x + 2y - 6 = 0$$

$2°$ $n < 0 \Rightarrow n = -\dfrac{6}{|m|}$

$$\frac{4}{m} + \frac{-3}{-\frac{6}{|m|}} = 1$$

$$\frac{4}{m} + \frac{|m|}{2} = 1 \quad / \cdot 2m$$

$$8 + m \cdot |m| = 2m$$

$$m \cdot |m| - 2m + 8 = 0 \qquad (2)$$

a) $m < 0$

$(2) \Rightarrow -m^2 - 2m + 8 = 0$

$$m^2 + 2m - 8 = 0$$

$$m_{1,2} = \frac{-2 \pm \sqrt{4 + 32}}{2} = \frac{-2 \pm 6}{2}$$

$$m_1 = \frac{-2 + 6}{2} = 2 \text{ (nije rješenje } (m < 0))$$

$$m_2 = \frac{-2 - 6}{2} = -4 \Rightarrow n = -\frac{6}{4} = -\frac{3}{2}$$

$$p \ \dots \ \frac{x}{-4} + \frac{y}{-\frac{3}{2}} = 1, \quad 3x + 8y + 12 = 0$$

b) $m > 0$

$(2) \Rightarrow m^2 - 2m + 8 = 0$

$$m_{1,2} = \frac{2 \pm \sqrt{4 - 32}}{2}$$

(nema rješenja)

Dva su rješenja: $3x + 2y - 6 = 0$ ili $3x + 8y + 12 = 0$.

ZADATAK 8.2.17

Odredi koeficijent k tako da površina trokuta što ga pravac $(k+1)x + ky - 18 = 0$ tvori s koordinatnim osima bude 27.

RJEŠENJE

$$(k+1)x + ky = 18 \quad / : 18$$

$$\frac{x}{\frac{18}{k+1}} + \frac{y}{\frac{18}{k}} = 1$$

$$\frac{|m \cdot n|}{2} = P$$

$$\frac{\left|\frac{18}{k+1} \cdot \frac{18}{k}\right|}{2} = 27$$

$$\frac{324 \cdot \left|\frac{1}{k+1} \cdot \frac{1}{k}\right|}{2} = 27$$

$$\left|\frac{1}{k+1} \cdot \frac{1}{k}\right| = \frac{1}{6}$$

$$\frac{1}{|k \cdot (k+1)|} = \frac{1}{6}$$

$$|k \cdot (k+1)| = 6 \qquad (*)$$

$1^\circ \quad k \in \langle -1, 0\rangle$

Iz $(*)$

$$-k(k+1) = 6$$

$$-k^2 - k - 6 = 0$$

$$k^2 + k + 6 = 0$$

$$k_{1,2} = \frac{-1 \pm \sqrt{1-24}}{2} \quad \text{(nema rješenja)}$$

$2^\circ \quad k \in \langle -\infty, -1\rangle \cup \langle 0, \infty\rangle$

Iz $(*)$

$$k(k+1) = 6$$

$$k^2 + k - 6 = 0$$

$$k_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2}$$

$$k_1 = \frac{-1-5}{2} = -3 \implies p_1 \ldots \ \frac{x}{-9} + \frac{y}{-6} = 1$$

$$k_2 = \frac{-1+5}{2} = 2 \implies p_2 \ldots \ \frac{x}{6} + \frac{y}{9} = 1$$

ZADATAK 8.2.18

Odredi koeficijent a tako da duljina odsječka pravca $3x + ay + 12 = 0$ između koordinatnih osi bude 5.

RJEŠENJE

$$3x + ay + 12 = 0$$

$$3x + ay = -12 \quad / : (-12)$$

$$\frac{x}{4} + \frac{y}{-\frac{12}{a}} = 1 \Rightarrow m = -4, \quad n = -\frac{12}{a}$$

$$d^2 = m^2 + n^2$$

$$5^2 = (-4)^2 + \left(-\frac{12}{a}\right)^2$$

$$25 = 16 + \frac{144}{a^2}$$

$$\frac{144}{a^2} = 9 \quad / \cdot a^2$$

$$144 = 9a^2 \quad / : 9$$

$$a^2 = 16$$

$$a = \pm 4$$

ZADATAK 8.2.19

Odredi a tako da odsječak pravca $ax + (1 - a)y + 4 = 0$ na osi Oy bude dvostruko dulji od njegovog odsječka na osi Ox.

RJEŠENJE

$$ax + (1 - a)y = -4 \quad / : (-4)$$
$$\frac{x}{-\frac{4}{a}} + \frac{y}{-\frac{4}{1 - a}} = 1$$
$$\frac{x}{-\frac{4}{a}} + \frac{y}{\frac{4}{a - 1}} = 1$$

$n = 2m$

$$\frac{4}{a - 1} = 2 \cdot \left(-\frac{4}{a}\right) \quad / \cdot a(a - 1)$$
$$4a = -8(a - 1)$$
$$4a = -8a + 8$$
$$12a = 8$$
$$a = \frac{2}{3}$$

$n = -2m$

$$\frac{4}{a - 1} = (-2) \cdot \left(-\frac{4}{a}\right) \quad / \cdot a(a - 1)$$
$$4a = 8(a - 1)$$
$$4a = 8a - 8$$
$$4a = 8$$
$$a = 2$$

Za $a = \frac{2}{3}$ imamo pravac $\frac{x}{-6} + \frac{y}{-12} = 1$, a za $a = 2$ pravac $\frac{x}{-2} + \frac{y}{4} = 1$.

ZADATAK 8.2.20

Odredi parametar a tako da duljine odsječaka pravca $ax + 3y - 1 = 0$ na koordinatnim osima budu jednake.

RJEŠENJE

Ako su duljine odsječaka jednake, onda je koeficijent pravca jednak 1 ili -1. Dakle, $a = 3$ ili $a = -3$.

$$ax + 3y - 1 = 0$$
$$ax + 3y = 1$$
$$\frac{x}{\frac{1}{a}} + \frac{y}{\frac{1}{3}} = 1 \Rightarrow m = \frac{1}{a}, \quad n = \frac{1}{3}$$
$$|m| = |n|$$
$$\left|\frac{1}{a}\right| = \left|\frac{1}{3}\right|$$
$$|a| = 3 \Rightarrow a = \pm 3$$

ZADATAK 8.2.21

Za koju je vrijednost realnog parametra a odsječak pravca $2x + ay - 5 = 0$ na osi apscisa dvostruko dulji od odsječka pravca na osi y?

RJEŠENJE

$$2x + ay - 5 = 0$$

$$2x + ay = 5 \quad / : 5$$

$$\frac{x}{\frac{5}{2}} + \frac{y}{\frac{5}{a}} = 1 \implies m = \frac{5}{2}, \quad n = \frac{5}{a}$$

$$|m| = |2n|$$

$|m| = |2n|$

$$\left|\frac{5}{2}\right| = 2 \cdot \left|\frac{5}{a}\right|$$

$$\frac{5}{2} = 10 \cdot \left|\frac{1}{a}\right|$$

$$|a| = 4 \implies a = \pm 4$$

ZADATAK 8.3.1

Odredi kut između pravaca:

1) $x - y + 5 = 0, \quad x + 2y - 1 = 0$

2) $2x - y + 1 = 0, \quad x + 3y - 2 = 0$

3) $x + y + 3 = 0, \quad x + 4y - 5 = 0$

4) $3x + 4y - 25 = 0, \quad 4x + 3y - 25 = 0$

5) $5x - y - 8 = 0, \quad 3x + 2y + 2 = 0$

6) $2x - 3y + 11 = 0, \quad 3x - y + 5 = 0.$

RJEŠENJE

1)

$$x - y + 5 = 0$$
$$x + 2y - 1 = 0$$

–

$$y = x + 5 \implies k_1 = 1$$
$$y = -\frac{1}{2}x + \frac{1}{2} \implies k_2 = -\frac{1}{2}$$

–

$$\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right| = \left|\frac{-\frac{1}{2} - 1}{1 - \frac{1}{2}\cdot 1}\right| = \left|\frac{-\frac{3}{2}}{\frac{1}{2}}\right| = 3 \implies \varphi \approx 71^\circ 34'$$

2)

$$2x - y + 1 = 0$$
$$x + 3y - 2 = 0$$

–

$$y = 2x + 1 \implies k_1 = 2$$
$$y = -\frac{1}{3}x + \frac{2}{3} \implies k_2 = -\frac{1}{3}$$

–

$$\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right| = \left|\frac{-\frac{1}{3} - 2}{1 - \frac{1}{3}\cdot 2}\right| = \left|\frac{-\frac{7}{3}}{\frac{1}{3}}\right| = 7 \implies \varphi \approx 81^\circ 52'$$

3)

$x + y + 3 = 0$
$x + 4y - 5 = 0$

–

$y = -x - 3 \implies k_1 = -1$

$y = -\frac{1}{4}x + \frac{5}{4} \implies k_2 = -\frac{1}{4}$

–

$$\text{tg}\,\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right| = \left|\frac{-\frac{1}{4} + 1}{1 + \frac{1}{4} \cdot 1}\right| = \left|\frac{\frac{3}{4}}{\frac{5}{4}}\right| = \frac{3}{5} \implies \varphi \approx 30^\circ 58'$$

4)

$3x + 4y - 25 = 0$
$4x + 3y - 25 = 0$

–

$4y = -3x + 25$
$3y = -4x + 25$

–

$y = -\frac{3}{4}x + \frac{25}{4} \implies k_1 = -\frac{3}{4}$

$y = -\frac{4}{3}x + \frac{25}{3} \implies k_2 = -\frac{4}{3}$

–

$$\text{tg}\,\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right| = \left|\frac{-\frac{4}{3} + \frac{3}{4}}{1 + \frac{4}{3} \cdot \frac{3}{4}}\right| = \left|\frac{-\frac{7}{12}}{2}\right| = \frac{7}{24} \implies \varphi \approx 16^\circ 15'$$

5)

$5x - y - 8 = 0$
$3x + 2y + 2 = 0$

–

$y = 5x - 8$
$2y = -3x - 2$

–

$y = 5x - 8 \implies k_1 = 5$

$y = -\frac{3}{2}x - 1 \implies k_2 = -\frac{3}{2}$

–

$$\text{tg}\,\varphi = \left|\frac{k_2 - k_1}{1 + k_1 k_2}\right| = \left|\frac{-\frac{3}{2} - 5}{1 - \frac{3}{2} \cdot 5}\right| = \left|\frac{-\frac{13}{2}}{-\frac{13}{2}}\right| = 1 \implies \varphi = 45^\circ$$

$2x - 3y + 11 = 0$
$3x - y + 5 = 0$

_

$3y = 2x + 11 = 0$
$y = 3x + 5$

_

6)
$y = \frac{2}{3}x + \frac{11}{3} = 0 \Rightarrow k_1 = \frac{2}{3}$
$y = 3x + 5 \Rightarrow k_2 = 3$

_

$$\mathrm{tg}\, \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right| = \left| \frac{3 - \frac{2}{3}}{1 + 3 \cdot \frac{2}{3}} \right| = \left| \frac{\frac{7}{3}}{3} \right| = \frac{7}{9} \Rightarrow \varphi \approx 37^\circ 52'$$

ZADATAK 8.3.2

Odredi unutarnje kutove trokuta što ga zatvaraju pravci $x - y + 3 = 0$, $x - 4y + 1 = 0$ i $4x + 2y - 7 = 0$.

RJEŠENJE

$x - y + 3 = 0$
$x - 4y + 1 = 0$
$4x + 2y - 7 = 0$

_

$y = x + 3$
$4y = x + 1$
$2y = -4x + 7$

_

$y = x + 3 \Rightarrow k_1 = 1$
$y = \frac{1}{4}x + \frac{1}{4} \Rightarrow k_2 = \frac{1}{4}$
$y = -2x + \frac{7}{2} \Rightarrow k_3 = -2$

_

$$\mathrm{tg}\, \alpha = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right| = \left| \frac{\frac{1}{4} - 1}{1 + \frac{1}{4} \cdot 1} \right| = \left| \frac{-\frac{3}{4}}{\frac{5}{4}} \right| = \frac{3}{5} \Rightarrow \alpha \approx 30^\circ 58'$$

$$\mathrm{tg}\, \beta = \left| \frac{k_1 - k_3}{1 + k_1 k_3} \right| = \left| \frac{1 + 2}{1 - 1 \cdot 2} \right| = 3 \Rightarrow \beta \approx 71^\circ 34'$$

$$\mathrm{tg}\, \gamma = \left| \frac{k_3 - k_2}{1 + k_2 k_3} \right| = \left| \frac{-2 - \frac{1}{4}}{1 - \frac{1}{4} \cdot 2} \right| = \left| \frac{-\frac{9}{4}}{\frac{1}{2}} \right| = \frac{9}{2} \Rightarrow \gamma \approx 77^\circ 28'$$

ZADATAK 8.3.3

Odredi unutarnje kutove trokuta kojem stranice leže na pravcima $x - y + 4 = 0$, $x + 2y = 0$ i $3x - y + 11 = 0$.

RJEŠENJE

$$x - y + 4 = 0$$
$$x + 2y = 0$$
$$3x - y + 11 = 0$$

–

$$y = x + 4$$
$$2y = -x$$
$$y = 3x - 11$$

–

$$y = x + 4$$
$$y = -\frac{1}{2}x$$
$$y = 3x - 11$$

–

$$k_a = 1, \quad k_b = -\frac{1}{2}, \quad k_c = 3$$

$$\operatorname{tg} \gamma = \operatorname{tg} \angle(a, b) = \left| \frac{k_b - k_a}{1 + k_a k_b} \right| = \left| \frac{-\frac{1}{2} - 1}{1 + 1 \cdot \left(-\frac{1}{2}\right)} \right| = \left| \frac{\frac{-3}{2}}{\frac{1}{2}} \right| = 3 \implies \gamma = 71°34'$$

$$\operatorname{tg} \alpha = \operatorname{tg} \angle(b, c) = \left| \frac{k_c - k_b}{1 + k_b k_c} \right| = \left| \frac{3 - 1}{1 + 3 \cdot 1} \right| = \left| \frac{2}{-4} \right| = \frac{1}{2} \implies \beta = 26°34'$$

$$\operatorname{tg} \beta = \operatorname{tg} \angle(b, c) = \left| \frac{k_c - k_a}{1 + k_a k_c} \right| = \left| \frac{3 + \frac{1}{2}}{1 + 3 \cdot \left(-\frac{1}{2}\right)} \right| = \left| \frac{\frac{7}{2}}{-\frac{1}{2}} \right| = 7 \implies \alpha = 81°52'$$